

# A Direct Method for Evaluating the Structural Needs of Flexible Pavements Based on FWD Deflections

Mario S. Hoffman, Ph.D.<sup>1</sup>

## Abstract

A direct and simple method (YONAPAVE) for evaluating the structural needs of flexible pavements is presented. It is based on the interpretation of measured FWD deflection basins using mechanistic and practical approaches. YONAPAVE estimates the effective Structural Number (SN) and the equivalent subgrade modulus independently of the pavement or layer thicknesses. Thus, there is no need to perform boreholes which are expensive, time consuming and disruptive to traffic. Knowledge of the effective SN and the subgrade modulus, together with an estimate of the traffic demand, allows for the determination of the overlay required to accommodate future needs. YONAPAVE simple equations can be solved using a pocket calculator, making it suitable for rapid estimates in the field. The simplicity of the method, and its independency of major computer programs, makes YONAPAVE suitable for estimating the structural needs of a road network using FWD data collected on a routine or periodic basis along the network roads. With increasing experience and confidence, YONAPAVE can be used as the basis for NDT structural evaluation and overlay design at the project level.

## Introduction

The 1993 AASHTO Guide for Design of Pavement Structures (1) presents three methods for determining the effective structural number ( $SN_{eff}$ ) of a conventional AC pavement. One of the methods, called the NDT method, is based on Nondestructive Testing (deflections) measurement and interpretation. This method assumes that the structural capacity of the pavement is a function of its total thickness and overall stiffness. The relationship between  $SN_{eff}$ , thickness and stiffness in the AASHTO Guide is:

$$SN_{eff} = 0.0045h_p \sqrt[3]{E_p} \dots [1]$$

Where:

$h_p$  = total thickness of all pavement layers above the subgrade, inches

$E_p$  = effective modulus of pavement layers above the subgrade, psi

The AASHTO guide recommends back-calculating  $E_p$  from deflection data using the two-layer linear elastic model (also known as the Burmister model).

The search of solutions to the problem of determining the effective Structural Number based on the interpretation of FWD deflections is not new (2, 3, 4). Most methods rely on the intrinsic relationship between measured deflection parameters, and layer coefficients or moduli and thickness of the pavement-subgrade system.

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<sup>1</sup> Technical Director and Partner, YONA, Engineering Consulting & Management Ltd., Haifa, ISRAEL

The idea of relating load-deformation responses (FWD deflection basins) to structural parameters (the Structural Number) is appealing. It conveys a fundamental relationship of classical mechanics. The choice of combining mechanistic postulates with AASHTO's Structural Number is somewhat controversial because of the empirical nature of SN (5). Empiricism, however, seems to be an integral part of pavement mechanics since the establishment of relationships such as  $M_R = 1,500 \text{ CBR}$  (6) (where  $M_R$  is the subgrade resilient modulus expressed in psi and CBR is the empirically defined California Bearing Ratio). This empirical relationship, which was first published 40 years ago, is still widely used everywhere.

Evaluating an existing pavement's SN is useful as it conveys structural adequacy or deficiency, and lends itself to determining structural needs. SN alone is not enough, though. A low SN is not necessarily bad. It depends on the subgrade support and the traffic demand. If subgrade support is high, and traffic demand is low, a low SN is all that is needed. Thus, the structural evaluation becomes useful only if subgrade support is evaluated together with SN.

A major drawback of the AASHTO approach, and of the methods derived from it, is their dependency on layer/pavement thicknesses. The same strong dependency on layer thicknesses exists with back-calculation methods that rely on FWD deflection basins for the determination of layer moduli using fitting techniques (7, 8).

While it could be argued that a few boreholes through the pavement layers could provide information on layer thicknesses, it is often found that  $h_p$  is ambiguous, heterogeneous, and difficult to determine, even for short pavement sections. This coring is also costly and time consuming, and cause disturbance to traffic flow.

Table 1 shows data of typical Israeli in-service road sections illustrating the difficulty in determining the value of  $h_p$ .

**Table 1: Layer Thickness and  $h_p$  in Israeli Road Sections Analyzed**

Road No.	Section Length (km)	Total No. Of Cores	Range of Values			Subgrade Type
			$h_{AC}$ (cm)	$h_{GR}$ (cm)	$h_p$ (cm)	
4	5.5	11	13-28	25-44	45-70	A-3
90	7.0	29	8-20	5-72	15-80	A-2-7
60	2.0	8	15-33	17-115	40-130	A-7-6
2	2.0	9	9-13	35-65	45-80	A-3
73	5.7	22	20-50	20-85	50-120	A-7-6
767	2.5	10	12-17	0-55	15-70	A-7-6
MB	1.0	8	8-18	22-90	35-110	A-2-4, A-7-6

Note to Table 1:  $h_{AC}$ =Thickness of AC layers;  $h_{GR}$ =Thickness of granular layers

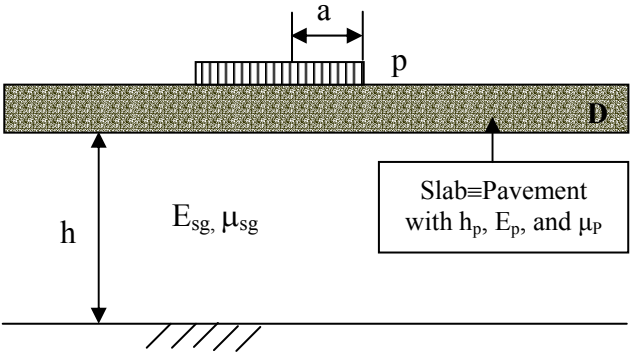
The value of  $h_p$  has a strong effect on the evaluation results. Using the scheme proposed by Rohde (2) it can be shown that  $h_p$  values in the range of 40 to 80 cm result in  $SN_{eff}$  estimates differing in 40% to 80% or more.

This paper presents a direct method (YONAPAVE) for determining  $SN_{eff}$  and the subgrade modulus of elasticity from measured FWD deflection basins that is independent of  $h_p$ . The method relies on the Hogg model of an infinite plate on an elastic subgrade of finite or infinite thickness. The subgrade E-values obtained with the proposed method highly correlate with the values determined using the MODULUS program (7, 9).

### Derivation of the YONAPAVE Method

A.H.A. Hogg reported on the analysis of a thin slab resting on an elastic foundation of infinite or finite depth in 1938 (10) and 1944 (11). Wiseman et al (12) described the applicability of the model for pavement evaluation. Hoffman (13) developed a solution to calculate deflection basins in the Hogg model under loads of any shape at any desired distance from the load center. Table 2 shows the model parameters and definitions.

**Table 2: Hogg Model Parameters and Definitions**

<p>Model Geometry</p>	
<p>Basic Model Parameters</p>	<p>Slab (pavement) Rigidity, <math>D = \frac{E_p h_p^3}{12(1 - \mu_p^2)} \dots [2]</math></p> <p>Characteristic Length, <math>l_0 = \sqrt[3]{\frac{D}{E_{sg}} * \frac{(1 + \mu_{sg})(3 - 4\mu_{sg})}{2(1 - \mu_{sg})}} \dots [3]</math></p> <p>Subgrade Modulus and Poisson's Ratio, <math>E_{sg}, \mu_{sg}</math></p>

Incorporating the values of  $\mu_p=0.25$  and  $\mu_{sg}=0.5$  into Equations [2] and [3], and doing proper algebraic substitutions, Equation [1] becomes:

$$SN_{eff} = 0.0182 l_0^3 \sqrt[3]{E_{sg}} \dots [4]$$

Where:

$l_0$  = Characteristic length, in cm,

$E_{sg}$  = Subgrade Modulus of Elasticity, in Mpa.

It is seen from Equation [4] that replacing the real pavement-subgrade system with the Hogg simplification permits the evaluation of the effective SN, as proposed by AASHTO, from the characteristic length and the modulus of elasticity of the subgrade. The effective SN is not a direct function of  $h_p$  anymore. The problem reduces to the determination of  $l_0$  and  $E_{sg}$  from FWD deflection basin interpretation.

## HOGG Deflection Basins

Figure 1 shows deflection basins calculated “loading” the Hogg model with a 6-inch radius circular plate representing the FWD geometry. The figure illustrates the variation of the deflection ratios  $D_r/D_0$  for deflections at any distance from the center relative to the central deflection for different values of  $l_0$ , and for a stiff bottom (bedrock) located at a depth of 20 times  $l_0$  ( $h/l_0=20$ ).

**Figure 1: Variation of  $D_r/D_0$  vs.  $l_0$  in the Hogg Model for  $h/l_0=20$ , FWD Loading**

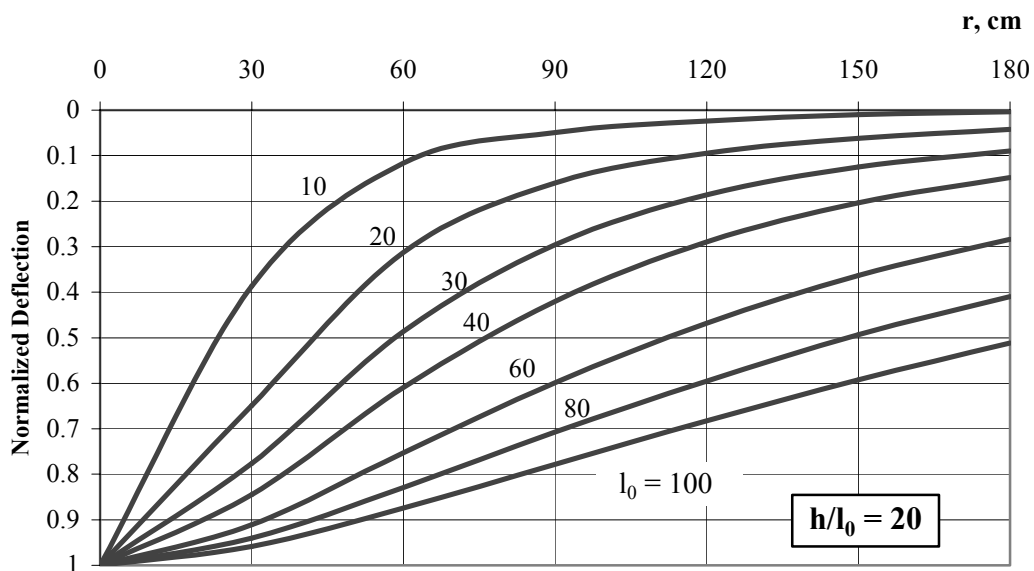


Figure 2 shows the variation of the deflection basin Area as a function of the characteristic length for the Hogg model with bedrock located at a depth of 10 times  $l_0$  ( $h/l_0=10$ ). The deflection basin Area is calculated from the following expression (12):

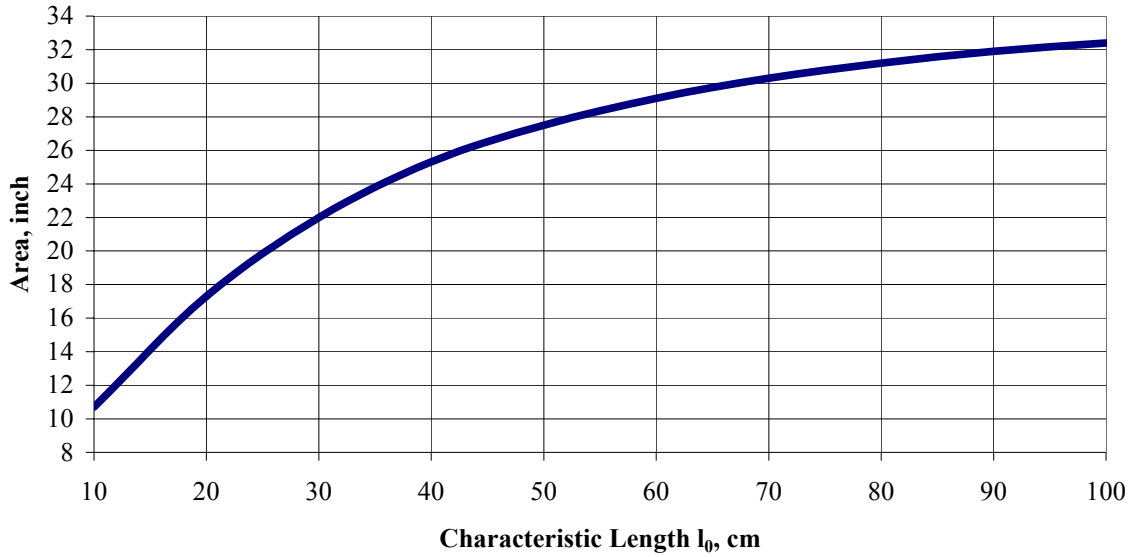
$$Area = 6(1 + 2 \frac{D_{30}}{D_0} + 2 \frac{D_{60}}{D_0} + \frac{D_{90}}{D_0}) \dots [5]$$

Where:

Area = Deflection basin Area, in inches,

$D_0, D_{30}, D_{60}, D_{90}$  = FWD deflections at  $r=0, 30, 60$  and  $90$  cm respectively.

**Figure 2: Deflection Basin “Area” vs.  $l_0$  in the Hogg Model for  $h/l_0=10$ , FWD Loading**



It is seen that the characteristic length is directly determined from the deflection basin Area for a chosen value of  $h/l_0$ . Similar Area vs.  $l_0$  curves can be developed for different values of  $h/l_0$ . It is also seen that the characteristic length is determined independently of the pavement thickness.

Figure 3 shows the variation of the maximum FWD deflection factor in the Hogg model as a function of the characteristic length for different values of  $h/l_0$ . The maximum deflection factor is defined as:

$$\text{MaximumDeflectionFactor} = \frac{D_0 E_{sg}}{pa} \dots [6]$$

Where:

$D_0$  = Maximum deflection under the FWD-12 inch diameter loading plate, in length units.

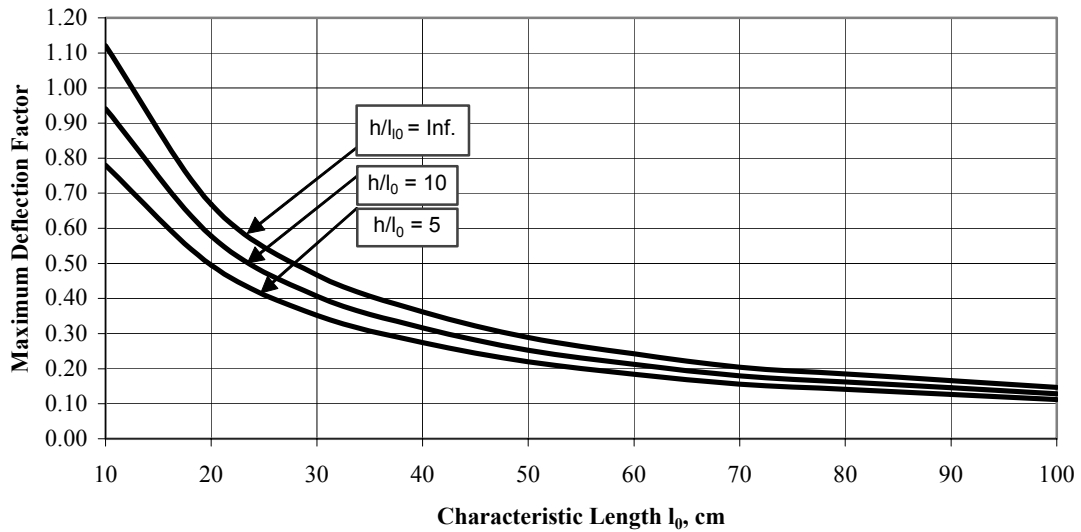
$E_{sg}$  = Modulus of Elasticity of the subgrade, in load/area units.

$p$  = Pressure on FWD loading plate, in same units as  $E_{sg}$ .

$a$  = radius of FWD loading plate, in same units as  $D_0$ .

Once  $h/l_0$  has been chosen, and  $l_0$  has been determined from a relationship similar to Figure 2, the maximum deflection factor can be determined from Figure 3. The modulus of elasticity of the subgrade is then calculated from the maximum deflection factor multiplying it by the actual pressure and radius of the FWD plate, and dividing it by the maximum measured FWD deflection. Having determined  $l_0$  and  $E_{sg}$ , the effective structural number of the pavement can be determined using equation [4].

**Figure 3: Maximum FWD Deflection Factor in the Hogg Model for different  $h/l_0$  Values**



### YONAPAVE Algorithms

Based on numerous comparisons between the subgrade E-moduli determined using the proposed method, and those determined using MODULUS, it was found that the best agreements are obtained when the depth of the bedrock in the Hogg model, i.e. the value of  $h/l_0$ , is determined as a function of the deflection basin Area. Using simple curve fitting techniques it is possible to express the relationship between the characteristic length and the deflection basin Area using an expression of the form:

$$l_0 = A \times e^{B \times \text{Area}} \dots [7]$$

Where:

$l_0$  = Characteristic length in cm,

Area = Deflection Basin Area, in inches,

A, B = Curve fitting coefficients as described in Table 3

**Table 3: Curve Fitting Coefficients for the Calculation of  $l_0$**

Range of Area Values, inch	$h/l_0$	A	B
Area $\geq 23.0$	5	3.275	0.1039
$21.0 \leq \text{Area} < 23.0$	10	3.691	0.0948
$19.0 \leq \text{Area} < 21.0$	20	2.800	0.1044
Area $< 19.0$	40	2.371	0.1096

In a similar way, it is possible to fit exponential curves for the determination of  $E_{sg}$  using an expression of the form:

$$E_{sg} = m \times \frac{p}{D_0} \times l_0^n \dots [8]$$

Where:

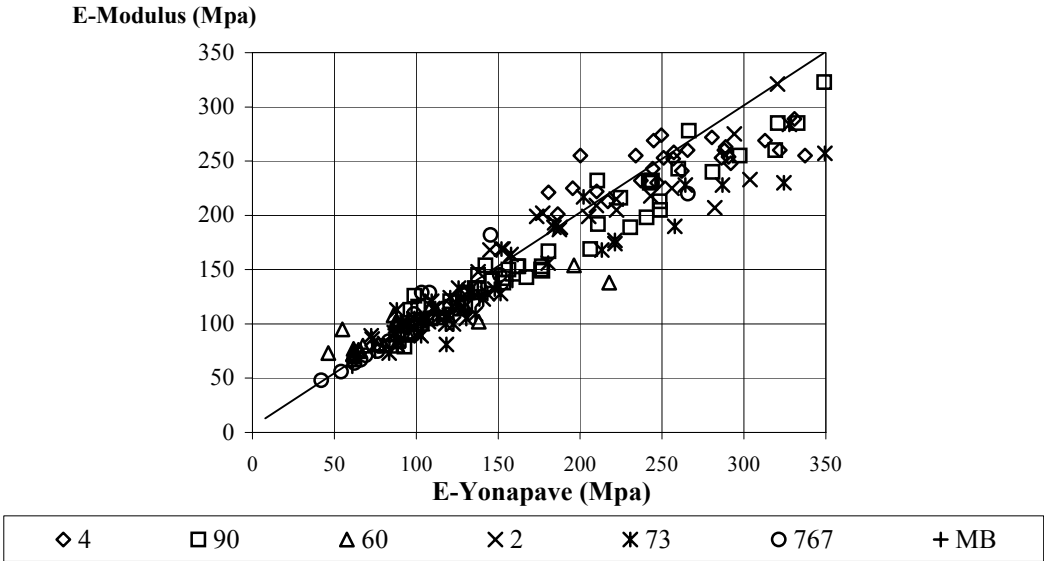
- $E_{sg}$  = Subgrade Modulus of Elasticity, in Mpa.
- $p$  = pressure on FWD testing plate, in kpa.
- $D_0$  = FWD Deflection under loading plate, in  $\mu\text{m}$ .
- $m, n$  = curve fitting coefficients as shown in Table 4

**Table 4: Curve Fitting Coefficients for the Calculation of  $E_{sg}$**

$h/l_0$	$m$	$n$
5	926.9	-0.8595
10	1,152.1	-0.8782
20	1,277.6	-0.8867
40	1,344.2	-0.8945

Figure 4 shows the fitness between  $E_{SG}$  values determined with MODULUS and YONAPAVE at in-service pavement sections in Israel (see Table 1). The figure shows that the E-values determined by both methods are in general good agreement, and reasonably follow the equality line. The best agreement is obtained for E-values below 200 MPa.

**Figure 4:  $E_{SG}$  MODULUS vs.  $E_{SG}$  YONAPAVE**



## Determination of $SN_{eff}$

Once the values of  $l_0$  and  $E_{SG}$  have been determined, as explained above, it is possible to calculate  $SN_{eff}$  using equation [4]. Because of the inherent characteristics of the Hogg model, where the pavement structure is modeled as a thin slab, and no deflections take place within the pavement structure, equation [4] under-predicts SN. Thus the following correction is proposed.

Based on numerous MODULUS back-calculation analyses using the best available thickness data for the existing pavement layers, a MODULUS derived SN was calculated together with the SN calculated with equation [4]. The MODULUS derived SN was based on the E-moduli backcalculated from FWD deflection basins using the scheme proposed in the AASHTO guide (1). This SN was adopted as the "correct" effective SN of the pavement.

A correlation between MODULUS derived SN values and SN values obtained using equation [4] renders a simple correction equation of the form:

$$\text{Corrected } SN_{eff} = 2 SN_{\text{Equation [4]}} - 0.5 \dots [9]$$

Equation [9] has a coefficient of determination of  $R^2 = 0.84$ . Thus the SN values obtained using Equation [4] should be corrected using Equation [9] to account for the Hogg model-thin slab related under predictions.

## Temperature Correction of $SN_{eff}$

Temperature has a direct effect on the asphalt layer modulus of elasticity. This effect is reflected in FWD's deflection basin parameters measured at different AC temperatures. The degree to which AC modulus of elasticity, and thus FWD deflections, are affected by temperature, depends on the AC composition, age, and degree of deterioration. The temperature effect on FWD deflections is further influenced by the AC layer thickness.

Based on FWD deflection basin measurements done at several Israeli flexible highway and airport pavements on the same summer day at different AC temperatures, it has been possible to establish typical ranges of AC temperature related effects (for AC layers of 10 cm or more), as indicated below:

- Typical variation of AC temperature at 5 cm depth: +60% (between morning and early afternoon)
- Typical variation of FWD maximum deflection under the loading plate for that AC temperature range: +20%
- Typical variation of deflection basin Area for that AC temperature range: -7%

For that range of AC temperatures, the MODULUS back-calculated E-moduli for a 3-layer characterization of those pavements have the following typical variations:

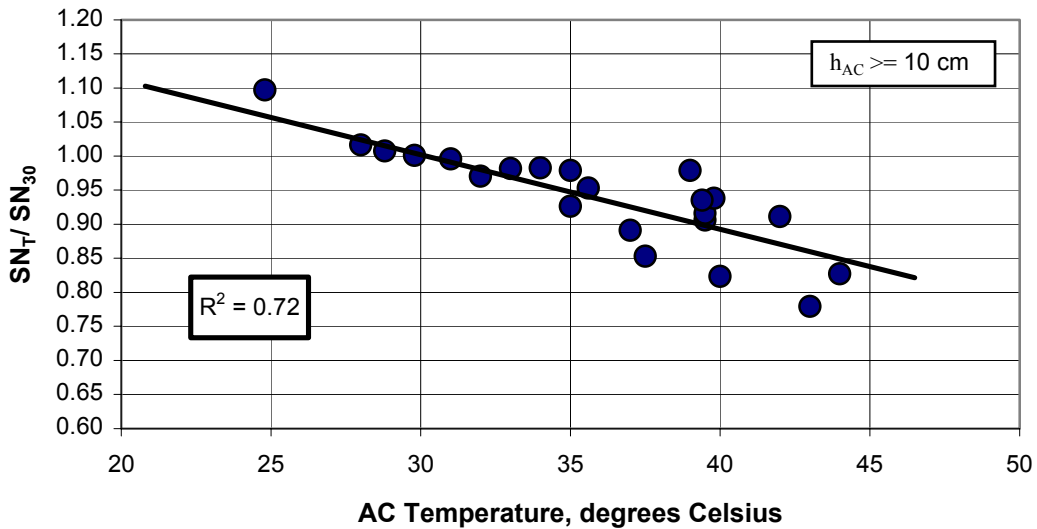
- Typical variation of  $E_{AC}$ : -50%
- Typical variation of  $E_{GR}$ : +10%
- Typical variation of  $E_{SG}$ : -10%

Where:  $E_{AC}$ ,  $E_{GR}$ , and  $E_{SG}$  are the modulus of elasticity of the asphalt concrete layer, the granular layer, and the subgrade, respectively. It is interesting to note that "fresh" AC mixtures tested in the laboratory would normally exhibit a variation in the resilient modulus of over 100% for the same range of temperatures (13). A discussion of the differences between field "layer" and laboratory "sample" behavior is beyond the scope of this paper, but it is generally observed that "layer" behavior in the field is less pronounced than "sample" behavior in the lab. This milder effect behavior has to do with the reciprocal effects among

the pavement layers and the subgrade in the field, which is difficult to reproduce and measure in the lab.

Figure 5 shows the variation of the effective Structural Number at any temperature relative to a base temperature of 30 °C versus the AC layer temperature measured at a depth of 5 cm. The SN-temperature values were computed using the YONAPAVE method at different Israeli highway and airport pavements where FWD deflection measurements were made on the same day at early morning to late afternoon temperatures. A base temperature of 30 °C was chosen to reflect Israeli climatic conditions.

**Figure 5: Variation of the Effective SN with the AC Temperature**



The relationship depicted in Figure 5 is of the form:

$$SN_T / SN_{30^{\circ}C} = 1.33 - 0.011T \dots [10]$$

Where:

SN<sub>T</sub> = Effective SN at any AC temperature

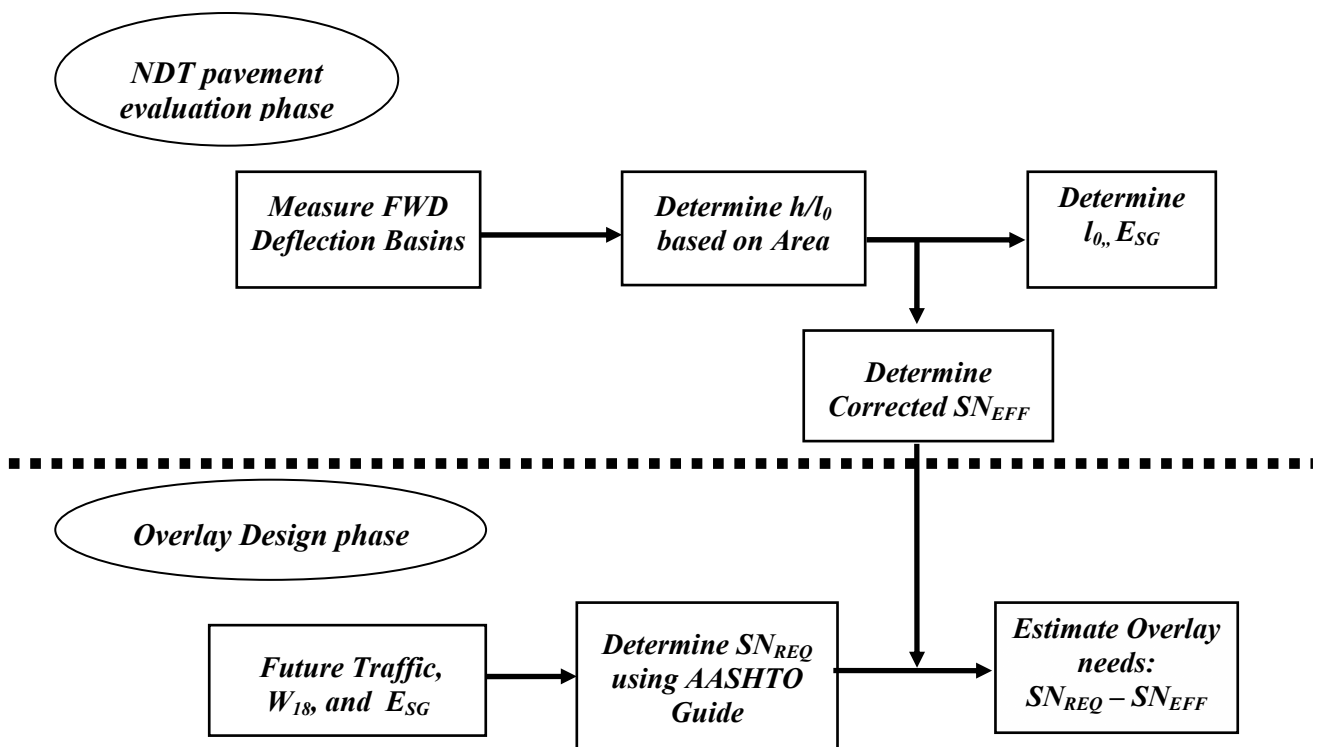
SN<sub>30°C</sub> = Effective SN at an AC base temperature of 30° C

T = Asphalt Concrete temperature in degrees C at a depth of 5 cm.

Equation [10] can be used to correct the YONAPAVE effective SN measured at different temperatures to a base AC temperature of 30° C. The equation was developed for AC temperatures in the range of 22 to 45° C. No extrapolations beyond that temperature range are recommended without field verification. The equation is applicable to AC layer thicknesses of 10 cm or more. For AC layers thinner than 10 cm there seems to be little effect of AC temperature on SN. From Figure 5 or Equation [10] it is possible to develop a temperature correction equation for a different base temperature other than 30° C.

### **Implementation of YONAPAVE for Structural Evaluation and Overlay Design**

The implementation of YONAPAVE for structural evaluation and overlay design is schematized in the flowchart below. It can be summarized in the following steps:



1. Perform FWD deflection basin measurements using a 45 to 75 KN load level (depending on the legal load limits in the network). Measure and record AC temperatures at a depth of 5 cm at regular time intervals (once every 1 to 2 hours).
2. Explore the need to divide the section into subsections based on the inspection of the maximum deflection and AREA plots and their variability along the section, on visual distress inspection, or other criteria.
3. Determine  $h/l_0$  based on the AREA values.
4. Compute  $l_0$  and  $E_{SG}$  using Equations [7] and [8], respectively.
5. Calculate the "uncorrected"  $SN_{eff}$  using Equation [4]. Correct the value of  $SN_{eff}$  using Equation [9].
6. Make SN temperature corrections using Equation [10].
7. Determine "design" values. Use a 30<sup>th</sup> percentile for  $E_{SG}$ , and a 10<sup>th</sup> to 30<sup>th</sup> percentile for corrected  $SN_{eff}$ . The recommended percentiles for  $SN_{eff}$  depend on the importance of the road analyzed. Use the lower percentiles for the major and most important roads and arteries.

The structural adequacy or the overlay needs can be determined using the following scheme:

1. Estimate future traffic demand in terms of 8.2 ton (18 kips) ESAL during the design period (10 to 20 years depending on budget or rehabilitation strategies).
2. Using the  $E_{SG}$  evaluated with YONAPAVE and the future traffic demand, determine the required SN based on the 1993 AASHTO Guide (1).
3. Compare the required SN with the evaluated corrected  $SN_{eff}$  to establish structural adequacy or strengthening needs. If the corrected  $SN_{eff}$  is higher than the required SN, there is no structural deficit in the pavement. If the corrected  $SN_{eff}$  is lower than

the required SN, it is possible to express the required strengthening in terms of AC overlay thickness using the following expression:

$$h_{AC} = (SN_{REQ} - SN_{EFF}) / \alpha \dots [11]$$

Where:

$h_{AC}$  = Thickness of AC overlay, inches

$\alpha$  = AC layer coefficient (use 0.44 as in AASHTO guide or other values)

Table 5 shows an example of the YONAPAVE method applied to the roads depicted in Table 1.

**Table 5: Example of YONAPAVE Results**

Road No.	Average $D_0$ , micron	Average AREA, inch	30 <sup>th</sup> Percentile $E_{SG}$ , Mpa	10 <sup>th</sup> /30 <sup>th</sup> Percentile Corrected $SN_{eff}$	Anticipated 10 year ESAL's	Range of $SN_{REQ}$	$\Delta SN$	Overlay $h_{AC}$ , cm
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
4	290	20.3	245	3.9	$33.1 \times 10^6$	3.3-3.6	0	0
90	390	19.0	164	3.1	$13.8 \times 10^6$	3.4-3.6	0.3-0.5	2-3
60	455	24.1	82	4.7	$11.0 \times 10^6$	4.1-4.5	0	0
2	340	20.3	186	3.4	$49.6 \times 10^6$	3.9-4.3	0.5-0.9	3-5
73	330	23.7	110	5.2	$7.4 \times 10^6$	3.5-4.2	0	0
767	665	21.1	86	3.6	$3.7 \times 10^6$	3.4-3.7	0-0.1	0-1
MB	640	20.7	92	3.2	$16.5 \times 10^6$	4.2-4.6	1.0-1.4	6-8

Notes to Table 5:

- 1) Same roads as in Table 1.
- 2) Average FWD Maximum Deflection under a 70 to 75 KN load.
- 3) Average deflection basin AREA.
- 4) 30<sup>th</sup> percentile  $E_{SG}$  for the road section analyzed.
- 5) 10<sup>th</sup> or 30<sup>th</sup> percentile of the corrected  $SN_{eff}$ .
- 6) Anticipated accumulated 8.2 ton (18kip) ESAL for a 10-year design period.
- 7) Range of required AASHTO-SN for 90% reliability, a serviceability loss of 2.0 or 1.5, and for  $E_{SG}$  values (column 4) and traffic estimates (column 6) as indicated.
- 8) Difference between the evaluated  $SN_{eff}$  (column 5) and the required  $SN_{REQ}$  (column 7), or zero if negative.
- 9) Required  $h_{AC}$  overlay for the range of  $\Delta SN$  obtained (column 8) and for an AC layer coefficient of 0.44 (for SN expressed in inches).

Table 5 shows that roads 4, 60, and 73 do not suffer from a structural deficit ( $\Delta SN$  equal 0). Existing distress in these roads can be caused by non-structural causes or the lack of timely maintenance, etc. Road 767 needs a minor structural overlay. Roads 90, 2 and MB exhibit a structural deficiency ranging from 2 cm to 8 cm of AC overlay needed. The results in Table

5 are in general good agreement with those obtained using the MODULUS or the AASHTO NDT interpretation approach.

## **Summary and Conclusions**

A simple and practical method (YONAPAVE) has been presented for evaluating the structural needs of flexible pavements. YONAPAVE solutions depart from the recommended NDT scheme of the 1993 AASHTO Guide to give an estimate of the effective Structural Number ( $SN_{EFF}$ ) and the subgrade Modulus of Elasticity.

YONAPAVE is based on the interpretation of measured FWD deflection basins using the HOGG model of a thin slab on an elastic subgrade to bypass the dependency of similar existing approaches on layer or pavement thickness. The independency of YONAPAVE on layer or pavement thickness is the major innovation relative to existing methods. A weak or cracked flexible pavement would normally result in low effective "slab" and subgrade evaluated parameters reflecting the actual condition of the road.

YONAPAVE has been calibrated with respect to more rigorous mechanistic formulations like MODULUS, including the incorporation of a rough rigid bottom at a finite depth. Based on this calibration, it has been shown that YONAPAVE and MODULUS produce similar estimates of the subgrade support for a wide range of subgrade types and moduli. The paper also presents an algorithm to correct the evaluated  $SN_{EFF}$  to a base AC temperature of 30 °C, and illustrates how to calculate the AC overlay thickness when the evaluated  $SN_{EFF}$  is below the required SN to meet future traffic demands.

YONAPAVE solutions have been reduced to simple equations that can be solved using a pocket calculator, making it suitable for rapid estimates of  $SN_{EFF}$  and subgrade moduli in the field. The simplicity of the method, together with its independency of layer or pavement thicknesses, make YONAPAVE suitable for handling large amounts of FWD data which is routinely of periodically collected on a road network within a Pavement Management System (PMS). The FWD deflection basins can be translated into  $SN_{EFF}$  values and subgrade moduli to monitor structural behavior with time. The method can be used to quantify and budget overlay needs at the network level. With increasing experience and confidence, YONAPAVE can be used as the basis for NDT structural evaluation and overlay design at the project level.

## **In Memoriam**

This paper is dedicated to the memory of Uzi Manor, a very talented and highly motivated civil engineer who participated in the analyses and the development of YONAPAVE algorithms, but a dark disease took him away so soon, and he was only 29 years of age.

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