CHAPTER 8

CALCULATION THEORY

Volume 2

Detailed in this chapter:

- the theories behind the program
- the equations and methods that are use to perform the analyses.

CONTENTS

CHAPTER 8 CALCULATION THEORY

8.1 GENERAL PILES

8.1.1 Vertical Analysis

This program uses procedures described in the *Foundations & Earth Structures,Design Manual 7.02,* published by Department of Navy, Naval Facilities Engineering Command.

8.1.1.1 Downward (Compression) Load Capacity Calculation

Ultimate downward capacity can be determined by the following equations:

$Q_{dw} = Q_{tip} + Q_{side}$

Where Q_{dw} = ultimate downward capacity

Q_{tip} = ultimate tip resistance

Q_{side} = ultimate side resistance

Ultimate tip resistance:

$Q_{tip} = A_{tip} \cdot q_{ult} = A_{tip} \cdot (N_q S_v + N_c)$

Where A_{tip} = area of pile tip

q_{ult}= ultimate end bearing pressure

S_v = vertical stress in soil (overburden pressure)

N_q = bearing factor for cohesionless soils. It is a function of friction shown in Table 8-1.

N_c = bearing factor for cohesive soils. It is a function of z/B (depth/width) shown in Table 8-2.

Table 8-1. Bearing Capacity Factor, N_q

f (Internal friction)	N_q (Displacement pile)	N_q (No-Displacement pile)
26	10	5
28	15	8
30	21	10
31	24	12
32	29	14
33	35	17

34	42	21
35	50	25
36	62	30
37	77	38
38	86	43
39	120	60
40	145	72

Table 8-2. Bearing Capacity Factor, N_c

z/B (Depth/Width)	N_c
0	6.3
1	7.8
2	8.4
3	8.8
4	9
>4	9

Ultimate side resistance:

$$Q_{side} = \Sigma\ S_f P_i\ \Delta l = \Sigma\ (f_0 + C_a)\ P_i\ \Delta l$$

Where S_f = side resistance

f_0 = skin friction of cohesionless soil

C_a = adhesion of cohesive soil

P_i = Perimeter of pile section

Δl = segment of pile

Skin friction of cohesionless soil:

$$f_0 = S_h \tan(d) = K_{down} \cdot S_v \cdot \tan(d)$$

Where $K_{down} = \dfrac{S_h}{S_v}$ or $S_h = K_{down} \cdot S_v$

S_v = vertical stress in soil

S_h = horizontal stress in soil

K_{down} = ratio of S_h/S_v which is defined in the table of Setup.

d = skin friction between soil and pile. It is a function of pile skin materials. For steel pile, $d = 20^{o}$-30^{o}. For concrete pile, $d = K_f\ \phi$. K_f is friction factor ranging from 0.1 to 1. K_f can be defined in the table of Setup.

Adhesion of cohesive soil:

$C_a = K_c \cdot K_a \cdot C$

Where C = shear strength of cohesive soil (cohesion)

K_c = adhesion factor ranging from 0.1 to 1, defined in the table of Setup.

K_a = Adhesion ratio, C_a/C, which is a function of C shown in Figure 8-1.

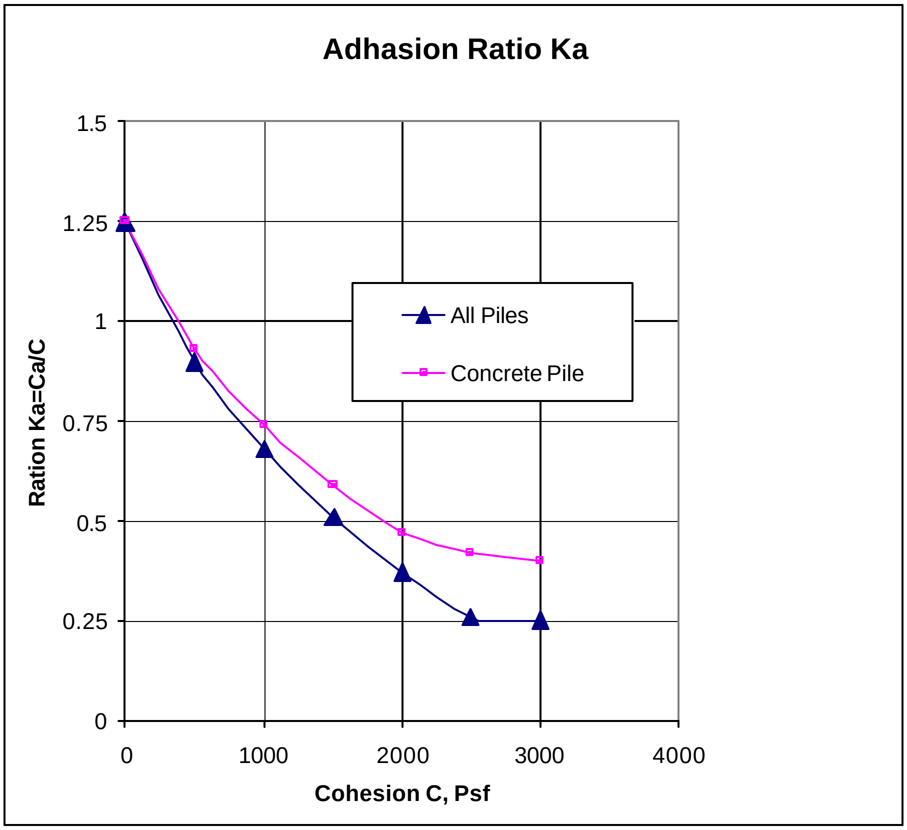


Figure 8-1. Adhesion Ratio, Ka

Limited Depth of side resistance and end bearing:

Experience and field evidence indicate that the side friction and end bearing increase with vertical stress ***S_v*** up to a limiting depth of embedment. Beyond this limiting depth (10D to 20D, D-pile width), there is very little increase in side friction and end bearing. Penetration Ratio, PR, is used to define the limiting depth. PR = 20 is commonly used for both side friction and end bearing. The values can be changed on the Advanced page.

$PR_{tip} = (z/D)_{tip}$, Penetration Ratio for calculation of end bearing.

$PR_{side} = (z/D)_{side}$, Penetration Ratio for calculation of side resistance.

Where z = depth

D = average pile width

The limitation of side friction and end bearing also can be expressed as absolute value for both cases. The values can be changed in the table of Advanced Page.

$q_{\_limit}$, Limit of end bearing pressure.

S_{f_limit}, Limit of sum of side friction and adhesion.

Allowable downward capacity can be determined by the following: equation:

$$Q_{allw\_d} = \frac{Q_{tip}}{FS_{tip}} + \frac{Q_{side}}{FS_{side}}$$

Where Q_{tip} = ultimate tip resistance

Q_{side} = ultimate side resistance

FS_{tip} = factor of safety for tip resistance, defined in the table of Advanced Page.

FS_{side} = factor of safety for side resistance in downward direction, defined in Advanced Page.

8.1.1.2 Zero Side Resistance

In some cases, a portion of the pile does not have contact with soils. For example, soils have gaps, or the pile passes through an underground basement or tunnel. Side resistance cannot be developed in this portion. Therefore the concept of zero friction can be used. It includes both zero friction and zero adhesion. Two zero-resistance zones can be input in the program.

8.1.1.3 *Zero Tip Resistance*

In special conditions, users do not want to include the tip resistance in pile capacity. These conditions include peat or soft soils at pile tip. Or the pile tip has a very sharp point. Users can include the depth of the pile tip in the zero resistance zones. For example, if the pile tip is at a depth of 35 feet, users can set a zero resistance zone from 35 to 36 feet. The tip resistance will be zero in the calculation.

8.1.1.4 *Negative Side Resistance*

Piles installed through compressive soils can experience “downdrag” forces or negative resistance along the shaft, which results from downward movement (settlement) of adjacent soil. Negative resistance results primarily from consolidation of soft deposits caused by dewatering or fill placement. The downdrag force is the sum of negative friction and adhesion. It does not include tip resistance. It only effects downward capacity, not uplift capacity. Two zero- and two negative-resistance zones can be input in the program. If the same zone is defined as both a zero-resistance and negative-resistance zone, the program considers the zone as a zero-resistance area.

Downdrag Force from Negative Friction:

$$Q_{neg} = K_{neg} \cdot \Sigma\ (S_f)\ P_i\ \Delta l = K_{neg} \cdot \Sigma\ (f_0 + C_a)\ P_i\ \Delta l$$

Where Q_{neg} = Downdrag force from negative side friction

K_{neg} = Negative side friction factor. It ranges from 0 to 1 depending on the impact of settlement of the soil to the pile shaft.

S_f = side resistance

f_0 = skin friction of cohesionless soil

C_a = adhesion of cohesive soil

Pi = Perimeter of pile section

Δl = segment of pile

8.1.1.5 *Maximum Settlement Calculation at Ultimate Vertical Resistance*

Based on Vesic’s recommendation (1977), the settlement at the top of the pile consists of the following three components:

Settlement due to axial deformation of pile shaft, X_s

$$X_s = \Sigma(Q_{tip} + Q_{side})\frac{\Delta l}{A'E}$$

Where Q_{tip} = tip ultimate resistance

Q_{side} = side ultimate resistance

Δl = pile segment

A’ = effective pile cross sectional area

E = modulus of elasticity of the pile

The equation is different from what shown in DM-7. This equation uses numerical integration, which is more accurate then the empirical equation in DM-7.

Settlement of pile point caused by load transmitted at the point, X_{pp}

$$X_{pp} = \frac{C_p Q_t}{B q_{ult}}$$

Where C_p = empirical coefficient depending on soil type and method of construction. It is defined in Table 8-3 below.

B = pile diameter

q_{ult}= ultimate end bearing pressure

Table 8-3. Typical Value of C_p for Settlement Analysis

Soil Type	Driven Piles	Drilled Piles
Sand	0.03	0.135
Clay	0.025	0.045
Silt	0.04	0.105

Settlement of pile point caused by load transmitted along the pile shaft, X_{ps}

$$X_{ps} = \frac{C_s Q_s}{L_e q_{ult}}$$

Where L_e = embedded depth

q_{ult}= ultimate end bearing pressure

Q_s = side resistance

$$C_s = (0.93 + 0.16\sqrt{\frac{z}{B}})C_p$$

Where z/B = depth / pile width

(Note: NAVY DM-7 has typo mistake in the equation)

Total settlement of a single pile, X_{total}

$X_{total} = (X_s+X_{pp}+X_{ps})$

8.1.1.6 Relationship Between Settlement and Vertical Load

Vertical load and settlement relation can be developed from t-z (side load vs shift movement) and q-w (bearing load vs base settlement) curves. The t-z curve represents the relation between side resistance and relative movement within soil and shaft. The t-z curve can vary at different depth and in different soils. The q-w curve represents the relation between tip resistance and base movement of the shaft.

t-z and q-w Relation

Generally, t-z and q-w relations require a considerable amount of geotechnical data from field and laboratory testings, which are not always available for engineers. AllPile uses the following procedures to determine the amount of settlement:

1. First, calculate ultimate side resistance and ultimate tip resistance of shaft using the methods introduced in 8.1.1.5.
2. Find relationships between settlement and load transfer ratios (developed resistance against ultimate resistance) using the corresponding charts in Fig 8-2 – 8-5
3. Integrate both side and tip resistances, as well as elastic compression of shaft body, to obtain total vertical resistance as a function of settlement.
4. From the relationships between settlement and load transfer ratios, we can develop t-z and q-w curve.

Typical settlement against load transfer ratios are shown in Figures 8-2 through 8-5 proposed by Reese and O'Neal (1988). Figure 8-2 and 8-3 represent the side load transfer ratio for cohesive soils and cohesionless soils/gravel respectively. Figure 8-4 and 8-5 represent the end bearing load transfer ratio for cohesive soils and cohesionless soils respectively.

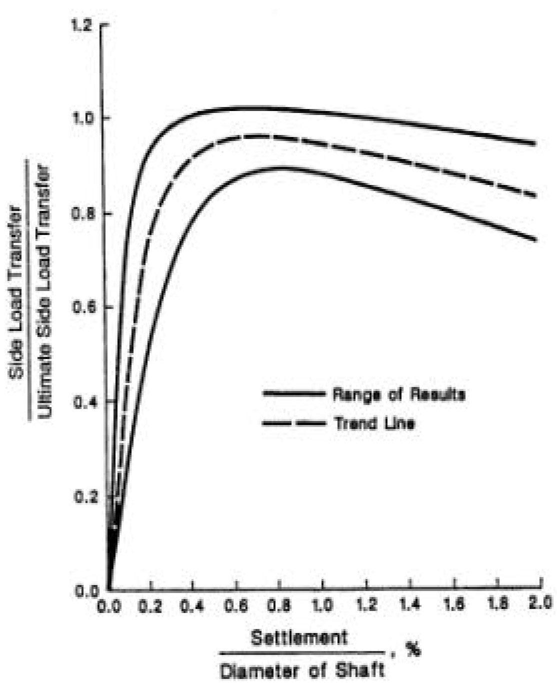


Figure 8-2. Normalized load transfer relations for side resistance in cohesive soil (Reese and O'Neill, 1989)

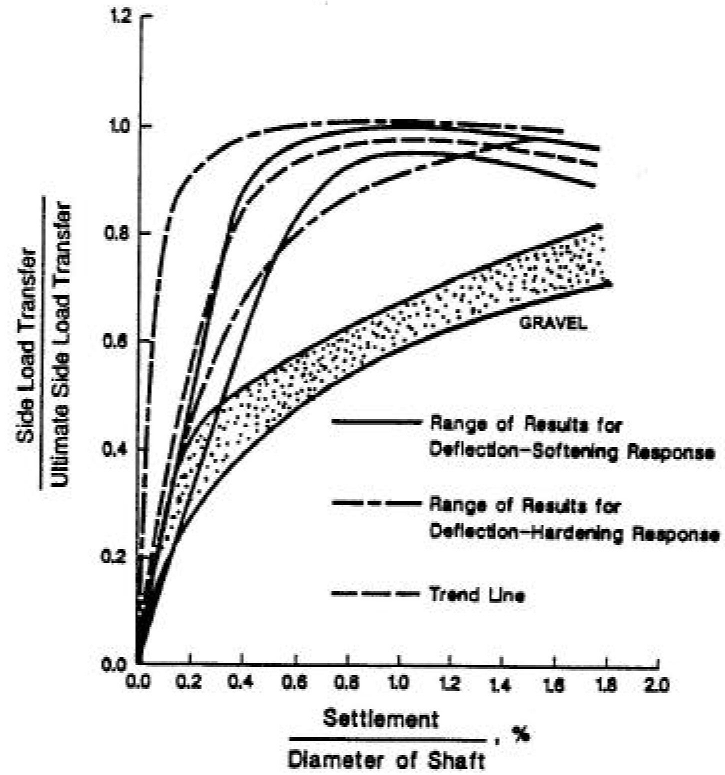
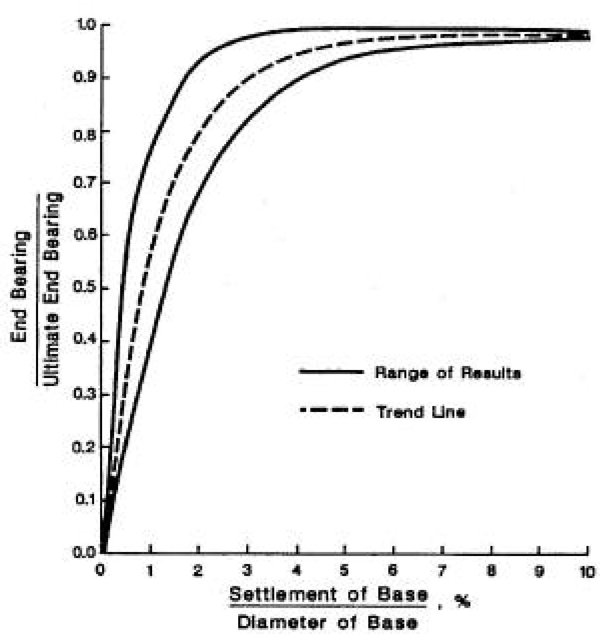


Figure 8-3 Normalized load transfer relations for side resistance in cohesionless soil (Reese and O'Neill, 1989)



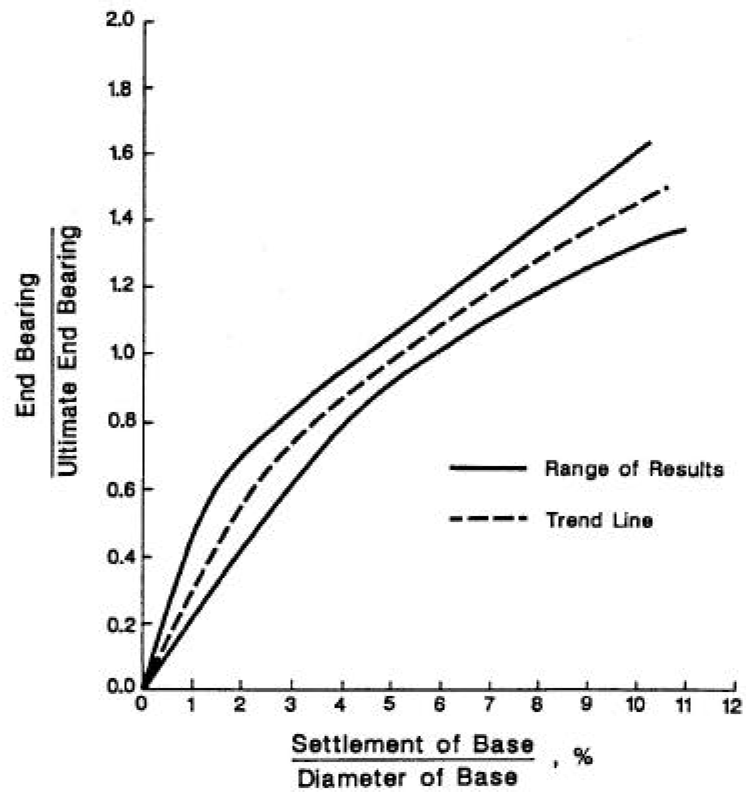


Figure 8-5. Normalized load transfer relations for base resistance in cohesionless soil (Reese and O'Neill, 1989)

Two Options for Settlement Analysis

In Advanced Page, AllPile provides two options for developing load-settlement relation.

Option 1:

The load transfer ratio is based on diameter of shaft (D_s) or base diameter of shaft (D_b) if it is different from the former, i.e. shafts with bell. This option is recommended for larger-size shafts.

Option 2:

The load transfer ratio is based on the calculated settlement from Vesic's method as described in Section 8.1.1.5. This option yields a closer match between settlement calculation of Vesic's method. It is recommended for smaller diameter piles.

Total, Side and Tip Resistance vs. Settlement

Figure 8-6 shows the vertical load is distributed in to side resistance and tip resistance. The chart from results of program shows that side resistance develops at small settlement, while tip resistance develops at large settlement. The ultimate value of the two cannot simply be added together. That is why tip resistance requires large Factor of Safety to get allowable capacity.

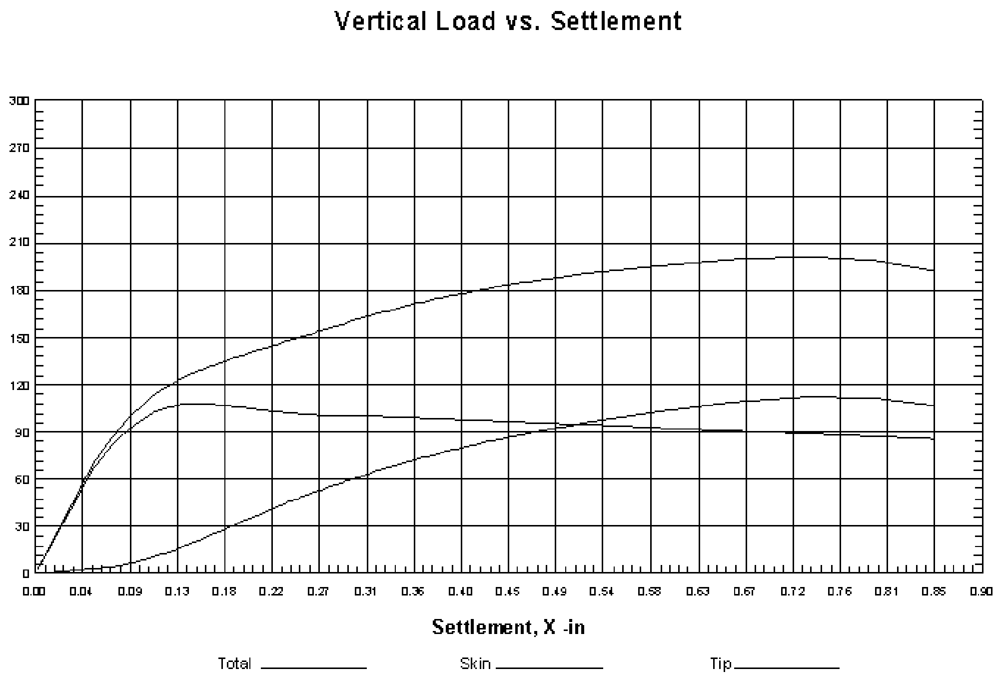


Figure 8-6. Total, Side and Tip Resistance vs. Settlement

Capacity at Allowable Settlement

AllPile provides two methods to determine Q_{allow}. One is defined by Factor of Saftey presented in Section 8.1.1.1. The other method is defined by allowable settlement.

Calculate Q_{allow} based on allowable settlement. Depending on the amount of allowable settlement X_{allow}, then back-calculate Q_{allow} based on the relationship between X_{allow} and load. X_{allow} can be defined on Advanced Page.

8.1.1.7 Uplift Load Capacity Calculation

Ultimate uplift capacity can be determined by the following equations:

$$Q_{up} = Q_w + Q_{side}$$

Where Q_w = weight of pile

Q_{side} = ultimate side resistance

$Q_w = \Sigma W_i \, \Delta l$

Where W_i = weight of pile section in unit length

Δl = segment of pile

$Q_{side} = \Sigma \, S_f \, P_i \cdot \Delta l = \Sigma \, (f_0 + C_a) \, P_i \, \Delta l$

Where S_f = side resistance

f_0 = skin friction of cohesionless soil

C_a = adhesion of cohesive soil

Δl = segment of pile

P_i = Perimeter of pile section

$f_0 = K_{up} \cdot S_v$ tand

Where $K_{up} = \dfrac{S_h}{S_v}$ or $S_h = K_{up} \cdot S_v$

S_v = vertical stress in soil

S_h = horizontal stress in soil

K_{up} = ratio of S_h/S_v which is defined in the table of Setup

d = skin friction between soil and pile. It is function of pile side materials. For steel pile, d = 20^o-30^o. For concrete pile, d = K_f? . K_f is friction factor ranging from 0.1 to 1. K_f can be defined in in the table of Setup.

$C_a = K_c \cdot K_a \cdot C$

Where C = shear strength of cohesive soil (cohesion)

K_c = adhesion factor ranging from 0.1 to 1, defined in the table of Setup.

K_a = Adhesion ratio, C_a/C, which is a function of C shown in Figure 8-1.

Allowable Uplift Capacity can be determined by following equations:

$$Q_{allw\_U} = \frac{Q_w}{FS_{\_w}} + \frac{Q_{side}}{FS_{\_up}}$$

Where Q_w = weight of pile

Q_{side} = ultimate side uplift resistance

$FS_{\_w}$ = factor of safety for pile weight, defined in the table of Advanced Page.

$FS_{\_up}$ = factor of safety for side resistance for uplift, defined in the table of Advanced Page.

8.1.1.8 Batter Shaft Capacities Calculation

The capacities of batter is from vertical capacities then adjusted by its batter angle:

$$Q_{batter} = \cos\alpha \cdot Q_{vertical}$$

Where α = Batter angle of shaft

Q = vertical capacities including downward and uplift

8.1.1.9 Group Vertical Analysis

In most cases, piles are used in groups as shown in Figure 8-7, to transmit the load to each pile. A pile cap is constructed over group piles. The analysis can be divided into four steps.

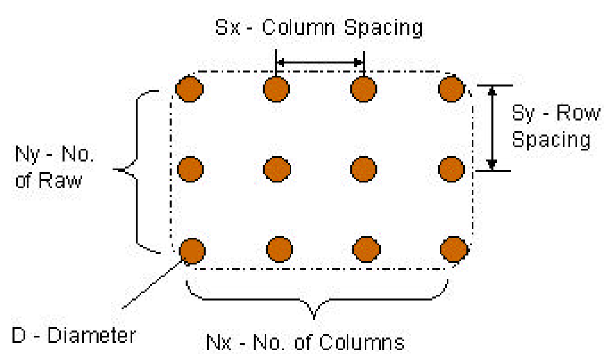


Figure 8-7 Group Pile for Vertical Analysis

Step 1. Calculate Capacity of Individual Pile, Q_{single}

Q_{single} can be calculated using the methods mentioned in above sections.
Q_{single} includes side resistance and tip resistance.

Step 2. Calculate Capacity of a Pile Block, Q_{block}

Q_{block} is calculated using single pile method including side and tip resistance. The block has the following dimensions:

$$B_x = (n_x-1) \cdot S_x + D$$

$B_y = (n_y\text{-}1) \cdot S_y + D$

L is the same as the length of each individual pile

Step 3. Calculate the Group Efficiency

$$h = \frac{Q_{block}}{nQ_{single}}$$

Where n = total number of pile. $n = n_x \cdot n_y$

$\mathbf{Q}_{single}$ = capacity of individual pile

$\mathbf{Q}_{block}$ = capacity of block pile

η = group efficiency

Step 4. Determine the Capacity of Group Pile, $\mathbf{Q}_{group}$

If $\eta = 1$, then $Q_{group} = n \cdot \mathbf{Q}_{single}$

If $\eta < 1$, then $Q_{group} = \mathbf{Q}_{block}$

8.1.1.10 Settlement Analysis for Group Pile

Suggested by Vesic (1969), the settlement for group pile can be estimated

$$X_{group} = X_{single} \cdot \sqrt{\frac{B'}{D}}$$

based on settlement of a single pile (DM7-7.2-209):

Where B' = smallest dimension between B_x and B_y (see Step 2 above)

D = diameter of a single pile

8.1.2 Lateral Analysis

AllPile directly uses COM624P calculation methods for lateral analysis. For details on COM624P, please refer to the FHWA publications, FHWA-SA-91-048, *COM624P – Laterally Loaded Pile Program for the Microcomputer, Version 2.0*, by Wang and Reese (1993). In that publication, Part I provides a User's Guide, Part II presents the theoretical background on which the program is based, and Part III deals with system maintenance. The appendices include useful guidelines for integrating COM624P analyses into the overall design process for laterally loaded deep foundations.

8.1.2.1 Lateral Deflection Calculation

Here is brief introduction to the program. COM624P uses the four nonlinear differential equations to perform the lateral analysis. They are:

$$EI\frac{d^4Y}{dZ^4} + Q\frac{d^2Y}{dZ^2} - R - P_q = 0 \qquad (1)$$

Where Q = axial compression load on the pile

Y = lateral deflection of pile at depth of Z

Z = depth from top of pile

R = soil reaction per unit length

E = modules of elasticity of pile

I = moment of inertia of the pile

P_q = distributed load along the length of pile

$$EI(\frac{d^3Y}{dZ}) + Q(\frac{dY}{dZ}) = P \qquad (2)$$

$$EI(\frac{d^2Y}{dZ^2}) = M \qquad (3)$$

Where P = shear in the pile

Where M = bending moment of the pile

$$\frac{dY}{dZ} = S_t \qquad (4)$$

Where S_t = slope of the elastic curve defined by the axis of pile

The COM624P program solves the nonlinear differential equations representing the behavior of the pile-soil system to lateral (shear and moment) loading conditions in a finite difference formulation using Reese's p-y method of analysis. For each set of applied boundary loads the program performs an iterative solution, which satisfies static equilibrium and achieves and acceptable compatibility between force and deflection (p and y) in every element.

Graphical presentations versus depth include the computed deflection, slope, moment, and shear in the pile, and soil reaction forces similar to those illustrated in Figure 8-8.

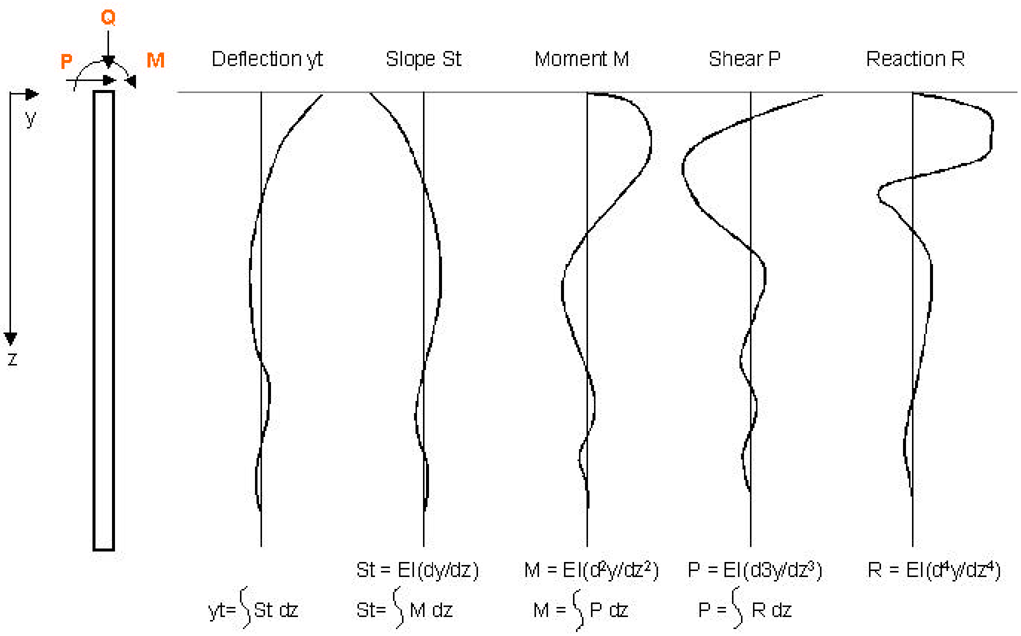


Figure 8-8 Graphical Presentation of AllPile Results

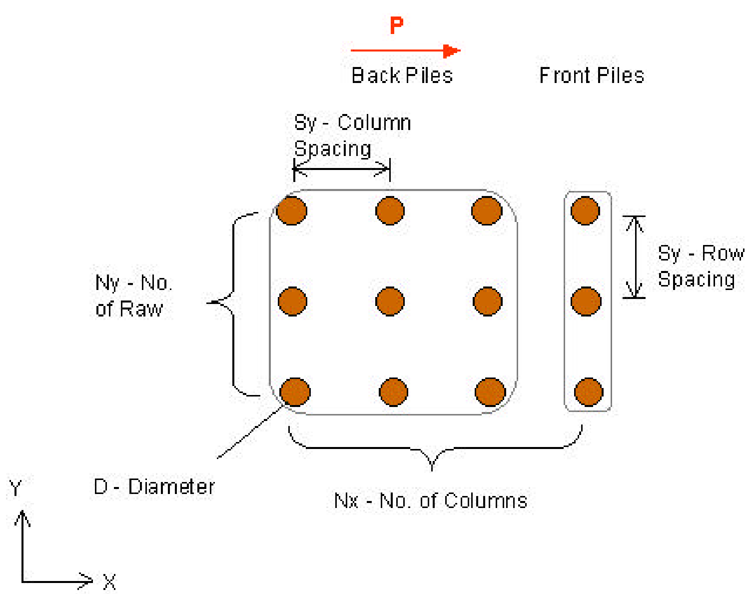


Figure 8-9 Group Pile for Horizontal Analysis

8.1.2.2 Group Lateral Analysis

Due to the group effect, the lateral capacity of individual piles can not be fully developed. Deduction factors are applied to the soil reaction, then lateral analysis is performed for individual piles.

Step 1. Calculate Deduction Factor R_{side} and R_{front}

Assuming the lateral load P is in X direction. Please note that R_{front} is not the same as R_{side}.

Table 8-4. Deduction Factor R_{front}

S_x	R_{front}
>8D	1
8D	1
6D	0.8
4D	0.5
3D	0.4
<3D	0.4

Table 8-5. Deduction Factor R_{side}

S_y	R_{side}
>3D	1
3D	1
2D	0.6
1D	0.3
<1D	0.3

Note: D = pile diameter Reference: *FHWA HI 97-013*

Step 2. Reducing Soil Lateral Resistance by Applying the Deduction Factors from Step 1

Combine the reduction factors and apply them to p-y curve for lateral analysis.

Step 3. Calculate P-yt (the Lateral Capacity and Deflection) of Each Individual Piles

Calculate the lateral capacity of each pile and get $P\text{-}Y_t$ curve for all piles.

Step 4. Get Lateral Capacity of group piles

After $P\text{-}Y_t$ curve for each pile is constructed. The P_{group} is the sum of P_{single} for individual piles at the same deflection under one pile cap.

$$P_{group} = \Sigma P_{single}$$

$$y_{group} = y_{single}$$

8.2 DRILLED SHAFT ANALYSES

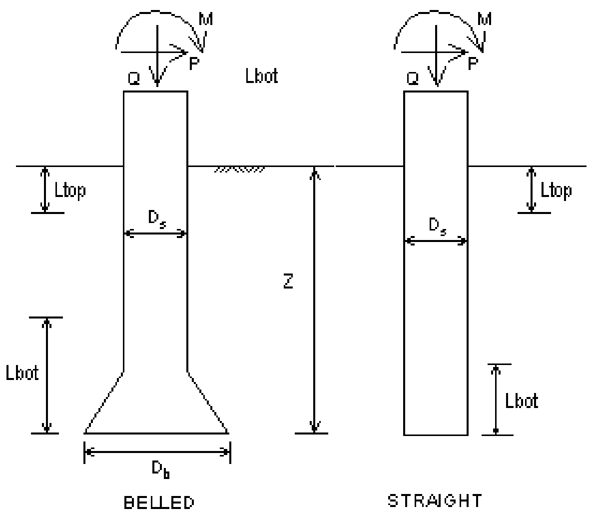


Figure 8-10. Drilled Shaft

Drilled shaft is normally used in deep foundation to transfer vertical load through weak soils to stronger soils or rocks at depth. Since it is often used to carry a relatively large vertical load over a good depth of soils, typical diameter of drilled shaft ranges from 4 ft (1.2 m) to 20 ft (6 m). In most cases, the aspect ratio of a drilled shaft, or its length divided by its diameter, should not exceed 30.

This program uses procedures described in the Drilled Shafts: Construction Procedures and Design Methods (FHWA-IF-99-025) published by FHWA in August 1999.

8.2.1 Vertical Analysis

8.2.1.1 Downward (Compression) Load Capacity Calculation

Ultimate downward capacity can be determined by the following equations:

$Q_{dw} = Q_{tip} + Q_{side}$

Where Q_{dw} = ultimate downward capacity

Q_{tip} = ultimate tip resistance

Q_{side} = ultimate side resistance

Ultimate tip resistance (Q_{tip}):

<u>Base in cohesive soils [$S_u \leq 0.25$ MPa (5,200psf)]</u>

$Q_{tip} = q_{ult}\ A_b$

Where q_{ult} = ultimate bearing pressure

A_b = base area

If Z (depth of base) ≥ $3D_b$ (diameter of base):

$q_{ult} = 9\ S_u$ [If $S_u \geq 96$kPa (1tsf)]

<u>or</u> $q_{ult} = Nc^* \cdot S_u$ [If $S_u < 96$kPa (1tsf)]

Where S_u = undrained shear strength below base

N_c* = modified bearing capacity factor for cohesive soils. It can be assumed to be a function of S_u in UU triaxial compression as shown in Table 8-6.

Table 8-6. Modified Bearing Capacity Factor, N_c*

S_u (Undrained Shear Strength)	**N_c*** (Bearing Capacity Factor)
24kPa (500psf)	6.5
48kPa (1000psf)	8.0
96kPa (2000psf)	9.0

If Z (depth of base) ≥ $3D_b$ (diameter of base):

$$q_{ult} = \frac{2}{3} + \left[1 + \frac{Z}{6D_b}\right] N_c * \cdot S_u$$

Base in cohesionless soils ($N_{SPT} \leq 50$)

In English unit: q_{ult} (kPa) = $57.5 \cdot N_{SPT}$

In Metric unit: q_{ult} (tsf) = $0.6 \cdot N_{SPT}$

Where N_{SPT} = blow count per 0.3m or 1ft of penetration in the Standard Penetration Test

Base in rocks [0.25MPa (2.5tsf) < S_u < 2.5MPa (25tsf)]

If embedment in rock ≥ $1.5D_b$ (diameter of base):

$q_{ult} = 5 \cdot S_u = 2.5 \cdot q_u$

Where q_u = unconfined compressive strength below base

**Attention*: The equation above are developed for drilled shafts embedded at least $1.5D_b$ (diameter of base) in good quality bedrock with RQD close to 100%. If rock is jointed or fractured, please consult geotechnical engineer for correct procedures to calculate tip resistance.

Ultimate side resistance (Q_{side}):

$$Q_{side} = \Sigma f_0 \cdot P_i \Delta l \cdot$$

Where f_0 = skin friction

Δl = segment of pile

P_i = perimeter of pile

<u>Shaft in cohesive soils [$S_u \le 0.25$ MPa (5,200psf)]</u>

$$f_0 = \alpha \cdot S_v$$

$\alpha = 0.55$ (for $S_u / P_a \le 1.5$)

$$a = 0.55 - 0.1\left(\frac{S_u}{P_a} - 1.5\right)$$

(for $1.5 \le S_u / P_a \le 2.5$)

Where α = shear strength reduction factor

P_a = atmospheric pressure = 101kPa or 2.12ksf

<u>Shaft in cohesionless soils ($N_{SPT} \le 50$)</u>

$$f_0 = \beta \cdot S_v'$$

In sand:

$\beta = 1.5 - 0.245\,[Z(m)]^{0.5}$ [If $N_{SPT} \ge 15$]

<u>or</u> $\beta = N_{SPT}/1.5 \cdot \{1.5 - 0.245\,[Z(m)]^{0.5}\}$ [If $N_{SPT} \ge 15$]

In gravelly sand or gravel:

$\beta = 2.0 - 0.15\,[Z(m)]^{0.5}$ [If $N_{SPT} \ge 15$]

<u>or</u> $\beta = N_{SPT}/1.5 \cdot \{1.5 - 0.245\,[Z(m)]^{0.5}\}$ [If $N_{SPT} \ge 15$]

Where β = empirical factor which varies with depth

S_v = effective vertical stress at depth Z

Z = depth where side resistance is calculated

<u>Attention</u>: - Z must be converted to meter before calculating β.

- Range of β: $0.25 \le \beta \le 1.2$

<u>Shaft in rocks [0.25MPa (2.5tsf) < S_u < 2.5MPa (25tsf)]</u>

$$f_0 = 0.65 \cdot P_a \left(\frac{q_u}{P_a}\right)^{0.5}$$

Where q_u = unconfined compressive strength at depth where side resistance is calculated

P_a = atmospheric pressure = 101kPa or 2.12ksf

8.2.1.2 Uplift Load Capacity Calculation

Ultimate uplift capacity can be determined by the following equations:

$Q_{up} = Q_w + Q'_{side} + Q'_b$

Where Q_w = weight of pile

Q'_{side} = ultimate side resistance against uplift

Q'_b = ultimate bell resistance against uplift (Q'_b is only calculated for belled shafts in cohesive soils)

$Q_w = \Sigma\ W_i \cdot \Delta l$

Where W_i = weight of pile section in unit length

Δl = segment of pile

$Q'_{side} = \Sigma\ k \cdot Q_{side}$

Where k = coefficient of uplift resistance

k = 1 (for cohesive soils)

k = 0.75 (for cohesionless soils)

k = 0.7 (for rocks)

Q_{side} = ultimate side resistance in compression in Section 8.2.1.1

*If a **belled** drilled shaft is used*

$Q'_b = N_u \cdot S_u \cdot A'_b$ (for cohesive soils only)

Where N_u = bearing capacity factor for uplift

= 3.5 Z/D_b or 9 (whichever is smaller)

Z = depth of drilled shaft

D_b = diameter of base/bell

S_u = undrained shear strength

A'_b = area of bell base minus area of shaft body ("Donut" area)

Attention: Belled shaft is not recommended for cohesionless soil and is too difficult to be constructed in rock layer. Therefore, Q'_b will not be considered in those two types of earth material.

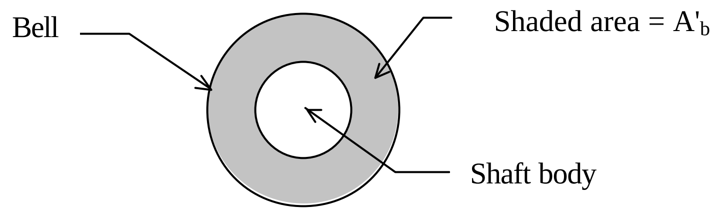


Figure 8-11 Top View of Donut Area

8.2.1.3 *Exclusion Zones*

According to the SHAFT manual, the exclusion zones do not contribute side resistance for drilled shaft as shown in Figure 8-10.

Exclusion zones in the calculation of ***Downward Capacity*** :

- For straight shafts: Top 5' and bottom one diameter of shaft
- For belled shafts: Top 5' belled section and one diameter of stem (D_s)

Exclusion zones in the calculation of ***Uplift Capacity*** :

- For straight shafts: Top 5'
- For belled shafts: Top 5', entire belled section and two diameter of stem (D_s) calculated from top of belled section

8.2.1.3 *Group Vertical Analysis*

In most cases, shafts are used in group as shown in Figure 8-12, to transfer the load to each shaft. A cap is constructed over group shafts. The analysis can be divided into four steps.

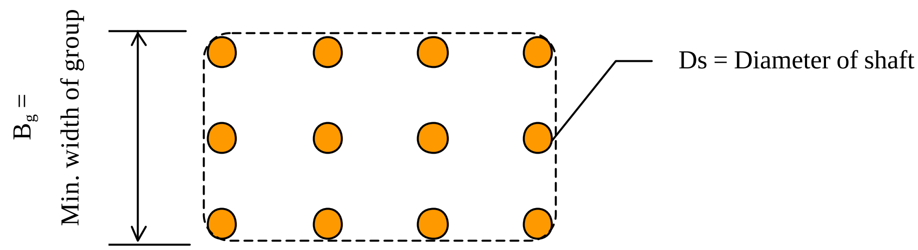


Figure 8-12 Group Shaft for Vertical Analysis

Step 1. Calculate Capacity of Individual Pile, Q_{single}

$\mathbf{Q}_{single}$ can be calculated using the methods mentioned in above sections.
$\mathbf{Q}_{single}$ includes side resistance and tip resistance.

Step 2. Calculate Minimum (Shortest) Dimension of Shaft Block, B_g

$B_g = (N_x-1) \cdot S_g + D_s$

Where N_x = number of shafts on the short side of the group

S_g = shaft spacing

D_s = diameter of drilled shaft

Step 3. Calculate the Group Efficiency, ?

$$h = \sqrt{\frac{B_g}{D_s}}$$

Where B_g = minimum width of shaft group

D_s = diameter of drilled shaft

Step 4. Determine the Capacity of Group Pile, Q_{group}

$Q_{group} = \eta \cdot Q_{single}$

8.2.2 Lateral Analysis

Lateral analysis for drilled shafts at single or group conditions are identical to that for drilled or driven piles. User can refer to Section 8.1.2 for the theories and the calculation procedures used in lateral analysis.

8.3 SHALLOW FOOTING ANALYSES

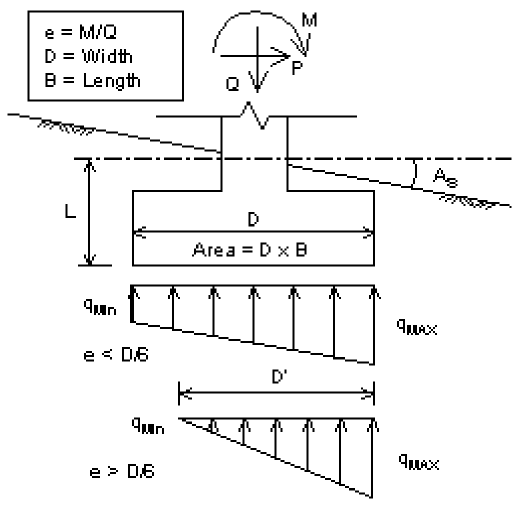


Figure 8-13. Shallow Footing

Shallow foundation is designed to transfer vertical load to relatively shallow depth through end bearing. Typical shallow foundations include spread footings, strip footings, and met footings. The bearing capacity of shallow foundation is influenced by a number of factors, which will be covered in the next section. On the other hand, shallow foundation is often subject to lateral loading or eccentricity. The stability of shallow foundation against eccentricity is controlled primarily by its ability to withstand overturning. *AllPile* uses procedures and recommendations given in *Principles of Foundation Engineering*, Brooks/Cole Engineering Division, Braja M. Das., 1984, as the primary references for shallow foundation analyses.

8.3.1 Vertical Analysis

8.3.1.1 Vertical (Compression) Load Capacity Calculation

Ultimate downward capacity (q_{ult}) can be determined by the following equation:

$$q_{ult} = c\ N_c\ s_c\ d_c\ i_c\ g_c + q\ N_q\ s_q\ d_q\ i_q\ g_q + 0.5\ \gamma\ D\ N_\gamma\ s_\gamma\ d_\gamma\ i_\gamma\ g_\gamma$$

Where c = cohesion

q = effective stress of soil at foundation base

γ = unit weight of soil

D = width or diameter of foundation

N = bearing capacity factors

s = shape factors

d = depth factors

i = load inclination factors

g = ground inclination factors

a.) Bearing Capacity Factors (N):

$$N_c = (N_q - 1)\cot f$$

$$N_q = \tan^2\left(45 + \frac{f}{2}\right)e^{p \tan f}$$

$$N_g = 2(N_q + 1)\tan f$$

b.) Shape Factor (s):

$$s_c = 1 + \left(\frac{B}{D}\right)\left(\frac{N_q}{N_c}\right)$$

$$s_q = 1 + \left(\frac{B}{D}\right)\tan f$$

$$s_g = 1 - 0.4\left(\frac{B}{D}\right)$$

Where, B = Length of footing

D = Width of footing

c.) Depth Factor (d):

For shallow foundations, in which embedment to footing width ratio (L/D) ≤ 1:

$$d_c = 1 + 0.4\left(\frac{L}{D}\right)$$

$$d_q = 1 + 2\tan f(1 - \sin f)^2 \frac{L}{D}$$

$$d_g = 1$$

For deeper foundations, in which L/D > 1:

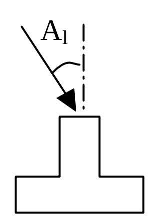
$$d_c = 1 + 0.4\tan^{-1}\left(\frac{L}{D}\right)$$

$$d_q = 1 + 2\tan f(1 - \sin f)^2 \tan^{-1}\left(\frac{L}{D}\right)$$

$$d_g = 1$$

Where, $\tan^{-1}$ (L/D) is in radius

d.) Load Inclination Factor (i):



$$i_c = i_q = \left(1 - \frac{A_l}{90°}\right)$$

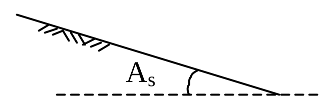
$$i_g = \left(1 - \frac{A_l}{f}\right)$$

Where, A_l is inclination of load in degree. $A_l = \tan^{-1}(P/Q)$ is in radius.

P = Shear Load, Q = Vertical Load

e.) Ground Inclination Factors (g) (Reference: *Foundation Design Principles & Practices*, Donald P. Conduto, p.176):

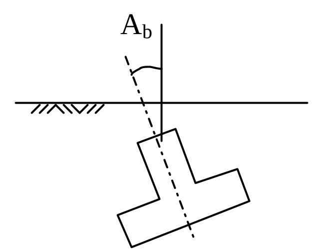
$$g_c = \left(1 - \frac{A_s}{147}\right)$$



$$g_c = g_g = (1 - 0.5 \tan A_s)^5$$

Where, A_s is angle of slope in degree.

f.) Battered Footing Reduction Factors (k_{bat}):



$$k_{bat} = \cos(A_b)$$

Where, A_b is angle of battered footing against

vertical axis in degree.

*Attention: Unlike other factors, k_{bat} is not applied to the equation of ultimate downward capacity (q_u) directly. It will be put into the calculation when the total ultimate downward capacity (Q_u) is calculated. Detail about Q_u is given below.

Total ultimate downward capacity (Q_u) represents the total bearing capacity against compression over the area of footing base. It can be determined by the following equations:

Net Ultimate Bearing Capacity (q_{net}):

$$q_{net} = q_u - q$$

Where, q_u = ultimate bearing capacity

q = overburden soil pressure

Total Ultimate Bearing Capacity (Q_{ult}):

$$Q_{ult} = (q_{net} \times k_{bat})\, A$$

Where, k_{bat} = battered footing reduction factor

A = base area of footing

Allowable downward capacity (Q_{allow}) can be calculated by the following equation:

$$Q_{allow} = \frac{Q_{ult}}{F.S.}$$

8.3.2 Capacity for Combined Loading

8.3.2.1 Vertical (Compression) Load (Q) Only

If there is only a vertical load, *Q*, without any lateral loading, i.e. shear loads and bending moment, the Factor of Safety of the shallow foundation can be calculated using the equation below:

$$F.S. = \frac{Q_{ult}}{Q.}$$

Where, Q_{ult} = ultimate bearing capacity

Q = total vertical load

Users can also check the ratio between Q and the allowable bearing capacity, Q_{allow}, to see if the shallow foundation is considered stable. If $Q > Q_{allow}$, the foundation is insufficient.

8.3.2.2 Vertical Load With Moment (Q + M)

Typical lateral loads on the foundation include bending moment (M) and shear load (P) as illustrated in the next diagram. In this section, we will study the procedures used to determine footing capacity against the combination of vertical load and bending moment (Q+M).

Eccentricity (e) will be generated by the moment and vertical load (see Figure 8-12):

$$e = \frac{M}{Q}$$

a.) If **e** £ **D/6**, the pressure on the foundation can be determined by:

$$q_{\max} = \frac{Q}{DB} + \frac{6M}{D^2 B}$$

$$q_{\min} = \frac{Q}{DB} - \frac{6M}{D^2 B}$$

Where, D = width of foundation base in lateral load direction

B = length of foundation base in the other direction

Reaction pressure at the base of the foundation distributed in a trapezoid pattern across the full width (D) of the foundation.

b.) If **e** > **D/6**, then:

$$q_{\max} = \frac{4Q}{4B(D - 2e)}$$

$$q_{\min} = 0$$

Reaction pressure at the base of the foundation is distributed in a triangular pattern across the effective width (D') of the foundation.

$$D' = D - 2e$$

Due to the distribution of reaction pressure, a new ultimate bearing capacity called, q_{ult}', has to be recalculated using the same procedures as mentioned in Section 8.3.1.1, but based on D' instead of D.

To calculate the Factor of Safety:

$$F.S. = \frac{q_{ult}'}{q_{\max}}$$

or

$$F.S. = \frac{Q_{ult}'}{Q}$$

Where, $Q_{ult}' = q_{ult}' \cdot D' \cdot B$

8.3.2.3 Vertical Load With Shear Load (Q + P)

The shear load (P) has two impacts to the shallow foundation calculation:

1. It generates load inclination – $A_l = \tan^{-1}(P/Q)$ – which affects vertical bearing capacity calculation (see Section 8.3.1.1).
2. Footing base sliding calculation becomes necessary. The sliding resistance (P_f) can be calculated by the following equation:

$$P_f = k_f (W + Q)$$

Where, k_f = base friction factor for cast-in-place foundation

k_f is close to $\tan\phi$ (ϕ = angle of internal friction)

k_f = 0.3-0.8 is recommended

W = weight of footing and the soil above

Factor of Safety against sliding can be calculated by:

$$F.S. = \frac{P_f}{P}$$

8.3.3 Settlement From Vertical Load

If only vertical load is applied to the shallow foundation, the elastic settlement (X_0) of the footing can be calculated using the equation below:

$$X_0 = \frac{Dq_0}{E_s}(1 - m^2)a$$

Where, q_0 = pressure under working load

μ = poisson ratio;

$$q_0 = \frac{Q}{Area} \quad or \quad q_0 = \frac{q_u}{F.S.}$$

= 0.3 is recommended for general soil conditions

E_s = Young's modulus

= $766N_{SPT}$ for cohesionless soils

= 375C for cohesive soils

Where, N_{SPT} = blow count over 12" of soil

C = undrained cohesion of soil

α = settlement factor for flexible foundation, which is a function of D/B (footing shape ratio)

[Note 1] X_0 is the elastic settlement at the center of a footing. If there is soft clay underneath the footing, consolidation settlement, which is time-dependent, should be considered. *AllPile* does not include calculation of consolidation settlement as it is not within the scope of the program.

[Note 2] AllPile assumes that a hard layer of soil, i.e. rock or intermediate geomaterials (IGMs) is in great depth from the base of footing. If H_a, the distance between footing base and hard soil, is over 4 times of footing width (D), the actual elastic settlement would not change considerably.

If H_a is less than 4D, the elastic settlement can be calculated based on the following equation:

$$X_0' = X_0 \frac{H_a}{4D} \qquad (H_a < 4)$$

Where, X_0' = actual elastic settlement when Ha < 4D

X_0 = elastic settlement based on H_a > 4D

H_a = distance between bottom of footing and hard soil

If user does not define H_a, *AllPile* will automatically search for the closest hard soil stratum with $N_{SPT} \geq 50$ based on user's input in the *Soil Property* page.

8.3.4 Rotation From Moment

The maximum settlement and rotation for a footing under both vertical and lateral loads can be determined by the following procedures:

1. Calculate eccentricity and effective width (D') based on section 8.3.2.2.
2. Determine the ultimate capacity (Q'_{ult}) under moment and vertical load from Section 8.3.2.2
3. Determine the Factor of Safety under both moment and vertical load

$$F.S._1 = \frac{Q_{ult}}{Q}$$

4. Calculate the ultimate capacity under vertical load only (see Section 8.3.2.1)

$$Q_{ult}(v)$$

5. Get Q_{allow} (v) under vertical load only

$$Q_{allow}(v) = \frac{Q_{ult}(v)}{F.S._1}$$

6. Calculate X_0 under Q_{allow} (v) based on Section 8.3.3

7. Determine the maximum settlement and rotation using the equations below:

$$X_{max} = X_0\left[1 + 2.31\left(\frac{e}{D}\right) - 22.61\left(\frac{e}{D}\right)^2 + 31.54\left(\frac{e}{D}\right)^3\right]$$

$$X_e = X_0\left[1 - 1.63\left(\frac{e}{D}\right) - 2.63\left(\frac{e}{D}\right)^2 + 5.83\left(\frac{e}{D}\right)^3\right]$$

$$R_t(Rotation) = \sin^{-1}\left[\frac{X_{max} - X_e}{\frac{D}{2} - e}\right]$$

Where, X_{max} = settlement at edges of footing

X_e = settlement under point of vertical load (vertical load may not apply to center of footing)

e = eccentricity

R_t = Rotation of Footing

*Note: These equations are only valid if $e/D \leq 0.4$

8.4 UPLIFT PLATE

Uplift plate is commonly used as a ground anchor to stabilize structure that is under shear load or moment. Due to its characteristics, uplift plate only provides uplift resistance against pulling, and has no bearing capacity. Uplift plate calculation can be divided into two modes:

- Shallow mode if L (= embedment) $\leq L_{cr}$
- Deep mode if $L > L_{cr}$

Where $\boldsymbol{L_{cr}}$ **= *critical depth*** in uplift resistance calculation

For cohesionless soils, L_{cr} is defined in Figure 8-14; For cohesive soils, L_{cr} can be determined using the following equation:

$$L_{cr} = D\,(0.107\,C_u + 2.5)$$

$$L_{cr} \leq 7D$$

Where C_u = undrained cohesion in kPa

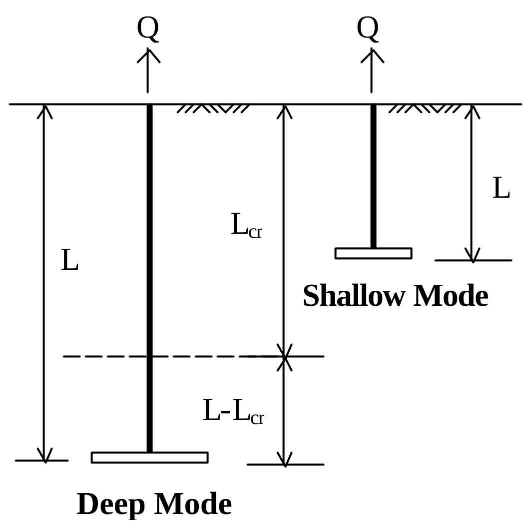


Figure 8-14 Critical Depth of Uplift Plates

8.4.1 Shallow Mode

For Cohesionless Soils

Uplift capacity (*$\boldsymbol{Q_{uplift}}$*) can be determined by the following equation:

$$Q_{uplift} = B_q \cdot A \cdot gL + W$$

Where, A = area of plate

W = weight of plate

B_q = breakout factor

$$B_q = 2\left(\frac{L}{D}\right)K_u'\tan f\left[m\left(\frac{L}{D}\right)+1\right]+1$$

Where, D = width of plate

K_u' = uplift factor, equal to 0.9 in general

ϕ = internal angle of friction of soil

m = shape factor coefficient, a function of ϕ and is defined in figure 8-15

For Cohesive Soils

Uplift capacity (Q_{uplift}) can be determined by the following equation:

$$Q_{uplift} = (C_u \cdot B_c + gL)A + W$$

Where, B_c = breakout factor, can be determined using the Figure 7-14 on page 78

C_u = undrained cohesion in kPa

A = area of plate

W = weight of plate

8.4.2 Deep Mode

For Cohesionless Soils

Uplift capacity (Q_{uplift}) in deep mode can be determined by the following equation:

$$Q_{uplift} = Q'_{plate} + Q'_{skin}$$

Where, Q'_{plate} = uplift capacity calculated in shallow mode

Q'_{side} = side resistance developed in the portion of $(L - L_{cr})$

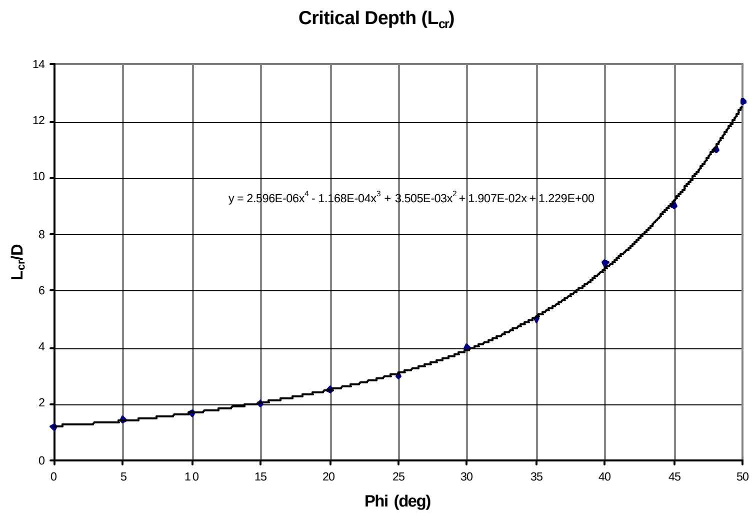


Figure 8-15 The relationship between critical depth (L_{cr}) and friction angle of soil (Phi)

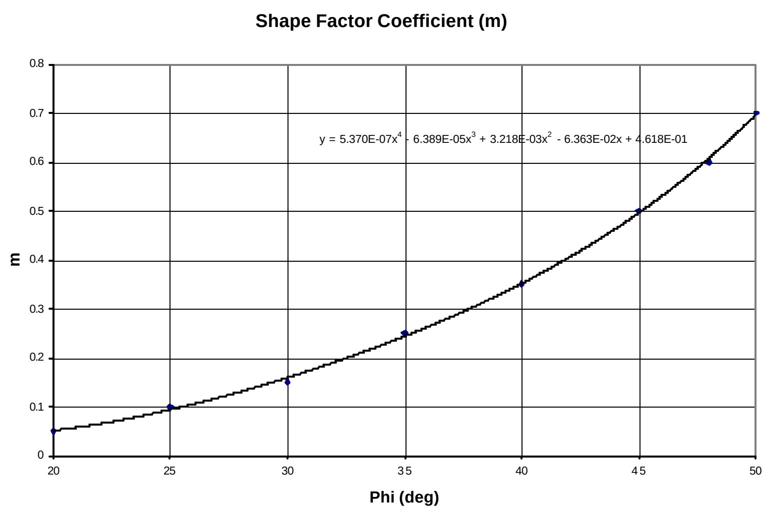


Figure 8-16 The relationship between shape factor coefficient (m) and friction angle of soil (Phi)

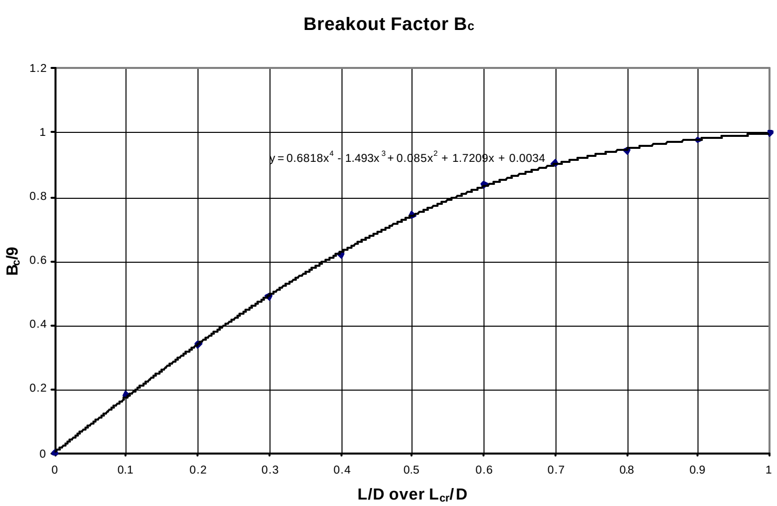


Figure 8-17 The relationship between breakout factor (B_c) and the ratio of embedment against critical depth

8.5 UPLIFT ANCHOR

Uplift anchor has the same function of an uplift plate, though it uses a completely different mechanism. Unlike uplift plate, which uses bearing capacity generated from its base plate against the soil mass on top of the plate to resist uplift force, uplift anchor generates the majority of its uplift resistance through adhesion and friction from its grouted section. An uplift plate can be divided into two portions.

- The top section, formed by uncovered steel bar which extends from the ground surface to the top of the grout, is typically called Free Length (L_f). Friction developed in this section is neglected.
- The bottom section is the grouted portion of the uplift anchor with a diameter of D. The total side resistance generated in this section is based on the adhesion of the grout and the bonded length (L_b).

The amount of adhesion is depended on grout pressure. The higher the grout pressure, the higher the adhesion that can be achieved from the bonded length. Post-grout also helps to generate higher adhesion.

$$Q_{uplift} = pD \cdot L_b \cdot C_a$$

Where, L_b = bonded length

C_a = adhesion input by user

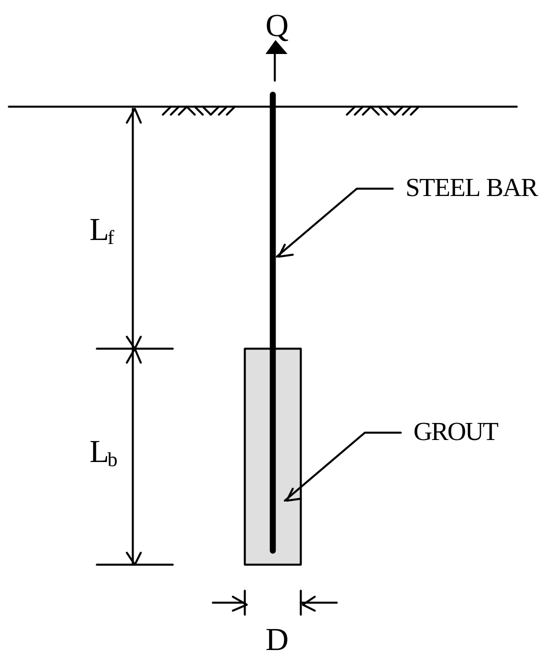


Figure 8.18 Uplift anchor

8.6 SOIL PARAMETERS AND CORRELATIONs

There are a number of references in the industry that present the correlations between soil parameters. The soil parameters function is useful if users only have a few parameters available and want to estimate the others to complete the calculation. However, one should bear in mind that these correlations are from various sources, statistical results of different soil types under different conditions. The actual value may be different from the estimate given by the correlation. Users should make their own judgement based on local experience and local soil conditions and adjust the value accordingly.

Following are the references used to form the soil correlation in the program:

Table 8-7. General Soil Parameters for Sand

Compactness	Symbol	Unit	Very Loose	Loose	Medium	Dense	Very Dense
SPT*	N_{SPT}	--	0-4	4-10	10-30	30-50	>50
Relative Density	Dr	%	0-15	15-35	35-65	65-85	85-100
Friction	ϕ	Deg	<28	28-30	30-36	36-41	>42
Unit Weight							
Moist	γ	pcf	<100	95-125	110-130	110-140	>130
Submerged	γ	pcf	<60	55-65	60-70	65-85	>75

*SPT -- Standard Penetration Test
Reference: Steel Sheet Piling Design Manual, USS, 1975, p.12

Table 8-8. General Soil Parameters for Clay

Consistency	Symbol	Unit	Very Soft	Soft	Medium	Stiff	Very Stiff	Hard
SPT	N_{SPT}	--	0-2	2-4	4-8	8-16	16-32	>32
UCS*	q_u	pcf	0-500	500-1000	1000-2000	2000-4000	4000-8000	>8000
Shear Strength	C_u	psf	0-250	250-500	500-1000	1000-2000	2000-4000	>4000
Unit Weight								
Saturated	γ	pcf	<100	100-120	100-130	120-130	120-140	>130

*UCS -- Unconfined Compressive Strength
Reference: Steel Sheet Piling Design Manual, USS, 1975, p.12

k and e50:

There are two parameters that are particularly important for lateral pile analysis—Modulus of Subgrade Reaction (k) and Soil Strain (E_{50}). Modulus of subgrade reaction is used in the equation $Es = k\,x$ in COM624 analysis, where *Es* is the secant modulus on a p-y curve and *x* is the depth below ground surface. The value of *k* describes the increase in *Es* with depth. Please note that the *k*-value is not the same as the coefficient of vertical subgrade reaction used to calculate elastic settlements of shallow foundations. It is also different from the coefficient of lateral subgrade reaction used in **elastic** pile analysis. (For more detail and example, please refer to NAVY DM7, 2-235. COM624 uses nonlinear differential analysis.) On the other hand, the soil strain E_{50} parameter is only applicable for clay soil and is obtained by either lab testing or by correlation. The input value E_{50} represents the axial strain at which 50% of the undrained shear strength is developed in a compression test. The following two tables demonstrate the correlation of *k* and E_{50} with other soil parameters for different soil type:

Table 8-9. Modulus of Subgrade Reaction (k) vs N_{SPT} for Sand

Compactness	Symbol	Unit	Loose	Medium	Dense
SPT	N_{SPT}	--	4-10	10-30	30-50
MSR*					
(Dry)	k	kN/m^3	6790	24430	61000
		pci	25	90	225
(Saturated)	k	kN/m^3	5430	16300	33900
		pci	20	60	125

*MSR -- Modulus of Subgrade Reaction
Reference: Handbook on Design of Piles and Drilled Shafts Under lateral Load, US Department of Transportation, 1984, p.64

Table 8-10. Modulus of Subgrade Reaction (k) and Soil Strain (E_{50}) vs N_{SPT} for Clay

Consistency	Symbol	Unit	Soft	Medium	Stiff	Very Stiff	Hard
SPT	N_{SPT}	--	2-4	4-8	8-16	16-32	>32
Shear Strength	C_u	kPa	12-24	24-48	48-96	96-192	192-383
		psf	250-500	500-1000	1000-2000	2000-4000	>4000
MSR*							
Static Loading	k	kN/m^3	8140	27150	136000	271000	543000
		pci	30	100	500	1000	2000
Cyclic Loading	k	kN/m^3	--	--	54300	108500	217000
		pci	--	--	200	400	800
Soil Strain	E_{50}	%	2	1	0.7	0.5	0.4

Reference: Lateral Load Piles, Lymon C. Reese, p.97

APPENDIX A   SYMBOLS AND NOTATIONS

Symbol	Description	English	Metric
S_v	Vertical stress in soil (overburden pressure)	ksf	kN/m^2
S_h	Horizontal stress in soil	ksf	kN/m^2
q_{ult}	Ultimate end bearing	ksf	kN/m^2
$S_f = f_0 + C_a$	Side resistance (ultimate), combination of skin friction and adhesion	ksf	kN/m^2
f_0	Skin friction from friction of soils (ultimate)	ksf	kN/m^2
C_a	Adhesion from cohesion of soils (ultimate)	ksf	kN/m^2
FS_{work}	Factor of safety at working load condition	--	--
FS_{side}	FS for side resistance in downward calculation	--	--
FS_{up}	FS for side resistance in uplift calculation	--	--
FS_{tip}	FS for tip resistance in downward calculation	--	--
FS_{w}	FS for weight of pile in uplift calculation	--	--
Q_{tip}	Vertical tip resistance	kip	kN
Q_{up}	Uplift ultimate capacity	kip	kN
Q_{dw}	Downward (compression) ultimate capacity	kip	kN
Q_{neg}	Load from negative friction	kip	kN
Q_{work}	Vertical work load or design load applied to pile	kip	kN
Q_{allw_u}	Allowable uplift capacity	kip	kN
Q_{allw_d}	Allowable downward capacity	kip	kN
Q_{group}	Vertical capacity of group pile	kip	kN
Q_{single}	Vertical capacity of single pile	kip	kN
Q_{plate}	Vertical uplift capacity of plate or bell	kip	kN
dz or dl	Pile segment	ft	m
K_{bat}	Factor for battered pile	--	--
R_t	Base rotation	degree	degree
R, R_{side} or R_{front}	Group reduction factor	--	--
y_t or y	Lateral deflection	in	cm
x	Vertical settlement	in	cm
L_{cr}	Critical depth in uplift analysis	ft	m

APPENDIX B   UNITS CONVERSIONS

English to Metric	**Metric to English**
1 ft = 0.3048 m	1 m = 3.281 ft
1 in = 2.54 cm = 25.4 mm	1 cm = 0.3937 in
1 lb = 4.448 N	1 mm = 0.03937 in
1 kip = 4.448 kN	1 N = 0.2248 lb
1lb/ft^2 = 47.88 N/m^2	1 kN = 224.8 lb = 0.2248 kip
1 kip/ft^2 = 47.88 kN/m^2 = 47.88 kPa	1 N/m^2 = 20.885 x 10^{-3} lb/ft^2
1lb/ft^3 = 0.1572 kN/m^3	1 kN/m^2 = 1 kPa = 20.885 lb/ft^2 = 20.885 x 10^{-3} kip/ft^2
1lb/in^3 = 271.43 kN/m^3	1 kN/m^3 = 6.361 lb/ft^3 = 0.003682 lb/in^3